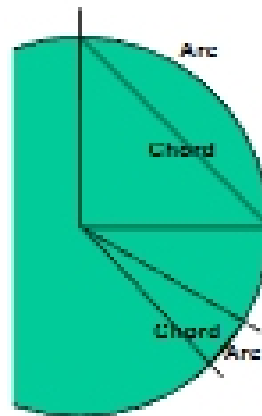


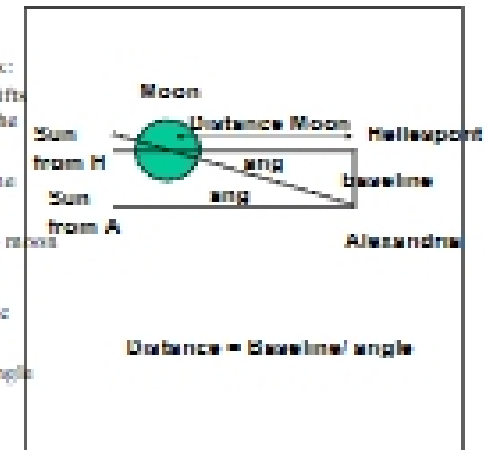
Small angle approximation

- Measure angles in radians
 - 2π radians = 360°
- Arc = radius \times angle
 - For entire circle,
 - arc = circumference = $2\pi R$
 - angle = 2π
- For small angles, arc is approximately equal to the chord.
 - **Chord = radius \times angle**
- Application
 - Baseline = Distance \times angle
 - $1000\text{km} = \text{Distance} \times 1/8^\circ$



Method of parallax

- Parts of triangle
 - Angle is due to parallax: moon in foreground shifts with respect to sun in the background.
 - One leg of triangle is the baseline.
 - Other leg is distance to moon
- Method of parallax.
 - Angle is the "parallaxic shift."
 - Distance = Baseline/angle



Difficulties

- Small angles are hard to measure
 - Naked eye $1/30^\circ = 1/3700$ rad = $6e-4$ rad = 600 grad
 - Modern telescope used under ideal conditions: 5 grad
 - Modern telescope with correction for atmospheric turbulence: 0.5 grad
 - Moon using Heliopolis & Alexandria
 - Angle = baseline/distance = $1000\text{km} / 400,000\text{km}$
 - = $1/400$ rad = 2500 grad ($1/7^\circ$)
 - Mars
 - Angle = baseline/distance = $1000\text{km} / 50,000,000\text{km}$
 - = 13 grad using Heliopolis & Alexandria.
- Need a reference nearby in the sky
 - Measuring with a reference on the ground is impossible.

Cassini & Richer 1672

- Angle=baseline/distance
- What baseline should C&R use to measure distance to Mars?



Cassini & Richer 1672

- Angle=baseline/distance
- What baseline should C&R use to measure distance to Mars?
- Cayenne-Paris baseline is 7000km.
 - Angle=baseline/distance=7000 km/80,000,000km = 90grad
 - Shift is 18 times width of the star with modern telescope



Cassini & Richer 1672

- We are pretending to be Jean Richer and Giovanni Cassini in 1672. We are measuring the distance to Mars by making observations from Paris and Cayenne.
 - Facing the screen, hold a pencil at arm's length. Without moving the pencil, look at it with your left and then your right eye. The pencil tip shifts with respect to the screen.
1. What corresponds to Mars?
 - A. Left eye or right eye
 - B. Tip of pencil
 - C. Something in the screen
 - D. The shift of the pencil tip with respect to the screen
 2. What corresponds to Paris?
 3. What corresponds to the star?
 4. What is proportional to the parallax angle?
 5. If Mars were closer, the shift is
 - A. bigger.
 - B. same.
 - C. smaller.
 6. If Paris and Cayenne were farther apart, the shift is
 - A. bigger.
 - B. same.
 - C. smaller.



For Fri

- We pretend to be M. Cassini. We will propose a grand plan to "Frame the World" to the king of France. How will the expedition to Cayenne determine the distance to every planet?