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2.004 Dynamics and Control II
Spring 2008

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Lecture 31¹

Reading:

- Nise: 10.1
- Class Handout: Frequency Response and Bode Plots

1 Sinusoidal Frequency Response (continued)

In Lecture 30 we saw that the steady-state response of a linear system with transfer function $H(s)$ to a sinusoidal input

$$u(t) = A \sin(\omega t + \theta)$$

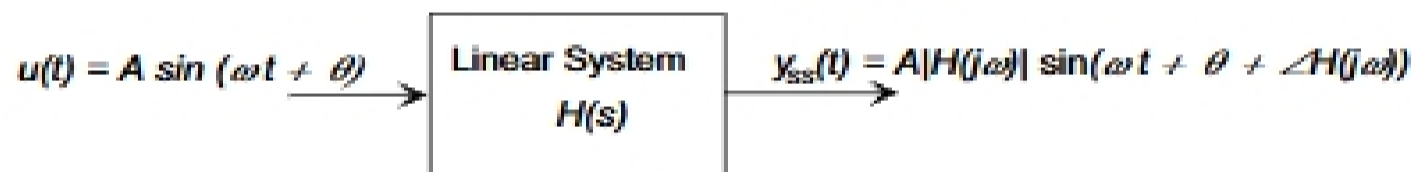
is

$$y_{ss}(t) = A |H(j\omega)| \sin(\omega t + \theta + \angle H(j\omega))$$

where

$$|H(j\omega)| = \sqrt{\Re^2\{H(j\omega)\} + \Im^2\{H(j\omega)\}}$$

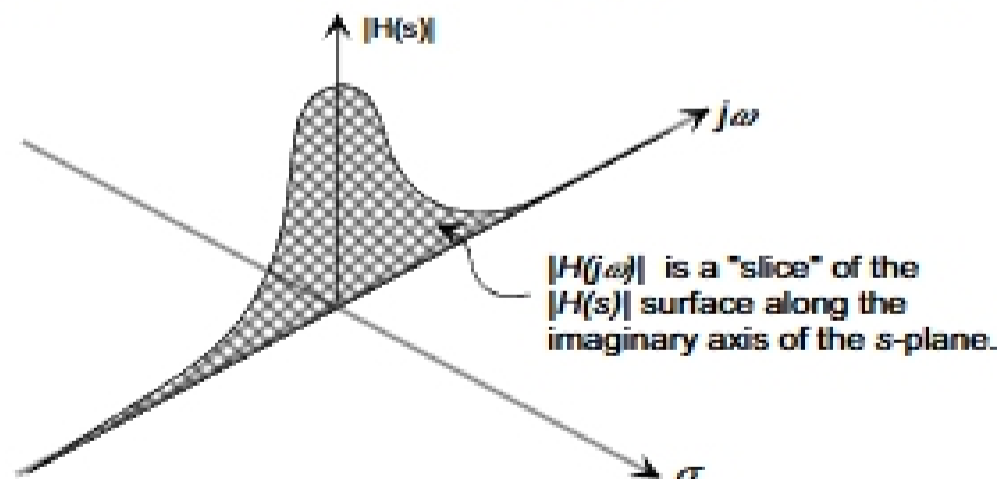
$$\angle H(j\omega) = \arctan\left(\frac{\Im\{H(j\omega)\}}{\Re\{H(j\omega)\}}\right)$$



We note that

$$H(j\omega) = H(s)|_{s=j\omega}$$

that is $H(j\omega)$ is $H(s)$ evaluated along the imaginary axis of the s -plane.



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1.1 The Frequency Response of Systems with Zeros

If a system has a transfer function

$$H(s) = \frac{N(s)}{D(s)}$$

the frequency response function is

$$H(j\omega) = \frac{N(j\omega)}{D(j\omega)}$$

For complex a and b , $|a/b| = |a|/|b|$ and $\angle(a/b) = \angle a - \angle b$, so that

$$|H(j\omega)| = \frac{|N(j\omega)|}{|D(j\omega)|} = \frac{\sqrt{\Re^2\{N(j\omega)\} + \Im^2\{N(j\omega)\}}}{\sqrt{\Re^2\{D(j\omega)\} + \Im^2\{D(j\omega)\}}} \quad (1)$$

$$\angle H(j\omega) = \angle N(j\omega) - \angle D(j\omega) = \arctan\left(\frac{\Im\{N(j\omega)\}}{\Re\{N(j\omega)\}}\right) - \arctan\left(\frac{\Im\{D(j\omega)\}}{\Re\{D(j\omega)\}}\right) \quad (2)$$

■ Example 1

Find and plot the frequency response of

$$H(s) = \frac{s + 5}{s + 10}$$

The frequency response function is

$$H(j\omega) = \frac{j\omega + 5}{j\omega + 10}$$

and

$$|H(j\omega)| = \frac{|N(j\omega)|}{|D(j\omega)|} = \frac{\sqrt{\omega^2 + 25}}{\sqrt{\omega^2 + 100}}$$
$$\angle H(j\omega) = \angle N(j\omega) - \angle D(j\omega) = \arctan\left(\frac{\omega}{5}\right) - \arctan\left(\frac{\omega}{10}\right)$$

The following MATLAB commands were used to plot the frequency response:

```
w=0:.2:100;  
sys=zpk(-5,-10,1)  
y=freqresp(sys,w);  
plot(w,squeeze(abs(y)))  
plot(w,squeeze(angle(y)))
```

which produced the following plots: