Shape Modeling with Front Propagation: A Level Set Approach

Ravikanth Malladi, James A. Sethian, and Baba C. Vemuri

***Abstract* — Shape modeling is an important constituent of computer vision as well as computer graphics research. Shape models aid the tasks of object representation and recognition. This paper presents a new approach to shape modeling which retains some of the attractive features of existing methods and overcomes some of their limitations. Our techniques can be applied to model arbitrarily complex shapes, which include shapes with significant protrusions, and to situations where no *a priori* assumption about the object's topology is made. A single instance of our model, when presented with an image having more than one object of interest, has the ability to split freely to represent each object. This method is based on the ideas developed by Osher and Sethian to model propagating solid/liquid interfaces with curvature-dependent speeds. The interface (front) is a closed, nonintersecting, hypersurface flowing along its gradient field with constant speed or a speed that depends on the curvature. It is moved by solving a "Hamilton-Jacobi" type equation written for a function in which the interface is a particular level set. A speed term synthesized from the image is used to stop the interface in the vicinity of object boundaries. The resulting equation of motion is solved by employing entropy-satisfying upwind finite difference schemes. We present a variety of ways of computing evolving front, including narrow bands, reinitializations, and different stopping criteria. The efficacy of the scheme is demonstrated with numerical experiments on some synthesized images and some low contrast medical images.**

***Index Terms* — Shape modeling, shape recovery, interface motion, level sets, hyperbolic conservation laws, Hamilton-Jacobi equation, entropy condition.**

I. Introduction

In this paper, we describe a modeling technique based on a level set approach for recovering shapes of objects in two and three dimensions from various types of image data. The modeling technique may be viewed as a form of active modeling such as "snakes" [15] and deformable surfaces [34] since, the model which consists of a moving front, may be molded into any desired shape by externally applied halting criteria synthesized from the image data. The "snakes" or deformable surfaces may be viewed as Lagrangian geometric formulations wherein the boundary of the model is represented in a parametric form. These parameterized boundary representations will encounter difficulties when the dynamic model embedded in a noisy data set is expanding/shrinking along its normal field [10] and sharp corners or cusps develop or pieces of the boundary intersect. By exploiting recent advances in interface techniques, our modeling technique avoids this Lagrangian geometric view and instead capitalizes on a related initial value partial differential equation. In this setting, several advantages are apparent, including the ability to evolve the model in the presence of sharp corners, cusps and changes in topology, model shapes with significant protrusions and holes in a seamless fashion, and extension to three dimensions in an extremely straightforward way.

A. Background

An important goal of computational vision is to recover the shapes of objects in 2D and 3D from various types of visual data. One way to achieve this goal is via model-based techniques. Broadly speaking, these techniques involve the use of a model whose boundary representation is matched to the image to recover the object of interest. These models can either be rigid, such as correlation-based template matching techniques, or nonrigid, as those used in dynamic model fitting techniques.

Shape recovery from raw data typically precedes its symbolic representation. Shape models are expected to aid the recovery of detailed structure from noisy data using only the weakest of the possible assumptions about the observed shape. To this end, several variational shape reconstruction methods have been proposed and there is abundant literature on the same (see [4], [27], [35], [38], [17] and references therein). Generalized spline models with continuity constraints are well suited for fulfilling the goals of shape recovery (see [6], [33]). Generalized splines are the key ingredient of the dynamic shape modeling paradigm introduced to vision literature by Kass et al [15]. Incorporating dynamics into shape modeling enables the creation of realistic animation for computer graphics applications and for tracking moving objects in computer vision. Following the advent of the dynamic shape modeling paradigm [15], [34], considerable research followed, with numerous application specific modifications to the modeling primitives, and external forces derived from data constraints [39], [18], [11], [24], [36], [37].

The final recovered shape in these schemes can depend on an initial guess which is reasonably close to the desired shape. One solution to this problem in the one-dimensional case has

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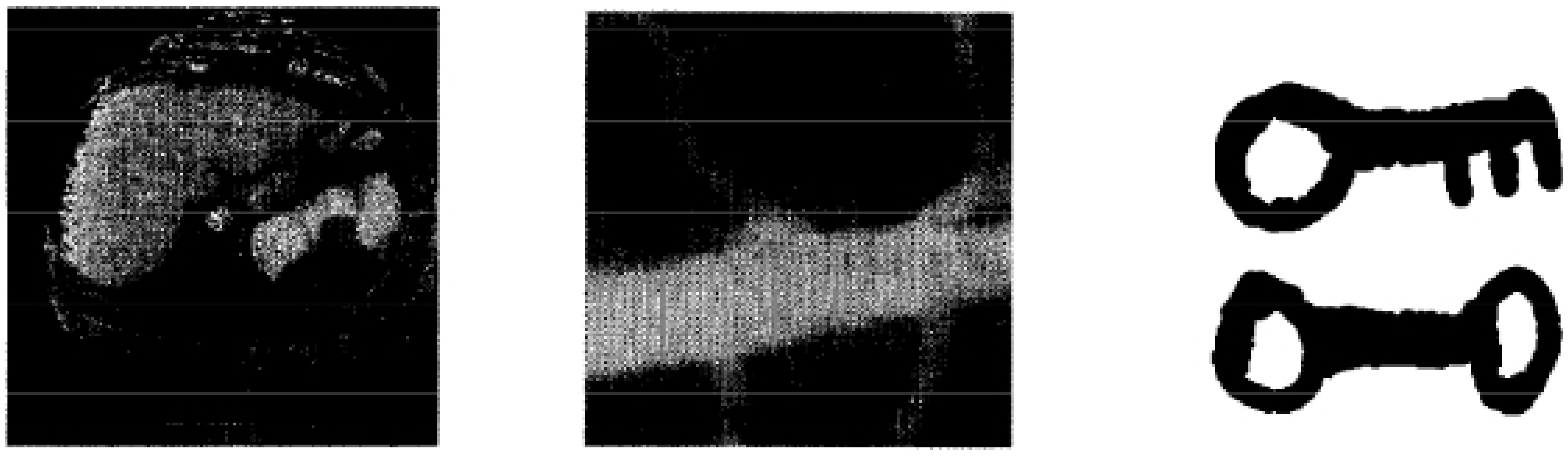

(a) CT image (b) DSA image (c) Shapes with holes

Fig. 1. Test bed for our topology-independent shape modeling scheme.

been presented by Amini et al [2]. They use a discrete form of dynamic programming to optimize the univariate variational problem.

The framework of energy minimization (snakes) has been used successfully in the past for extracting salient image contours such as edges and lines by Kass et al [15]. To make the final result relatively insensitive to the initial conditions, Cohen [10] suggested the use of an inflation force which makes the snake behave like an edge seeking active model. Although the inflation force prevents the curve from getting trapped by isolated spurious edges, the active contour model cannot be made to extrude through any significant protrusions that a shape may possess (see Fig. 1(b)) without resorting to cumbersome resampling techniques. In this paper, we present a technique which overcomes this problem and accurately models bifurcations and protrusions in complex shapes. Most existing shape modeling schemes require that the topology of the object be known before the shape recovery can commence. It is, however, not always possible to specify the topology of an object prior to its recovery. For example, an important concern in object tracking and motion detection applications is topological change resulting from tracking the positions of object boundaries in an image sequence through time. During their evolution, these closed contours may change connectivity and split, thereby undergoing a topological transformation. One such example is the splitting of cell boundary in a sequence of images depicting cell division. A heuristic criterion for splitting and merging of curves in 2D which is based on monitoring deformation energies of points on the elastic curve has been discussed in [26]. In the context of static problems, more recently, particle systems have been used to model surfaces of arbitrary topology [32]. Here, particles can be added and deleted dynamically to enlarge, and trim the surface respectively.

The schemes described in this paper offer a new approach to some of the above problems. To begin, the convergence to the final result is relatively independent of the shape initialization. The algorithm allows branches to sprout automatically as the front moves. The scheme described in this paper can be applied where no *a priori* assumption about the object's topology is made. A single instance of our model, when presented with an image having more than one shape of interest (see Fig. 1(c)), has the ability to split freely to represent each shape [19], [20]. We show that by using our approach, it is also possible to extract the bounding contours of shapes with holes in a seamless fashion (see Fig. 13).

Our method is inspired by ideas first introduced in Osher and Sethian [23], [29], which grew out of work in Sethian [28], to model propagating fronts with curvature-dependent speeds. Two such examples are flame propagation and crystal growth, in which the speed of the moving interface normal to itself depends on transport terms modified by the local curvature. The challenge in these problems is to devise numerical schemes for the equations of the propagating front which will accurately approximate these highly unstable physical phenomena. Osher and Sethian [23] achieve this by viewing the propagating surface as a specific level set of a higher-dimensional function. The equation of motion for this function is reminiscent of an initial value "Hamilton-Jacobi" equation with a parabolic right-hand side and is closely related to a viscous hyperbolic conservation law.

In our work, we adopt these level set techniques to the problem of shape recovery. To isolate a shape from its background, we first consider a closed, nonintersecting, initial hypersurface placed inside (or outside) it. This hypersurface is then made to flow along its gradient field with a speed $F(K)$, where K is the curvature of the hypersurface. Unknown shapes are recovered by making the front adhere to the object boundaries. This is done by synthesizing a speed term from image data which acts as a halting criterion. Finally, we note that a separate study also applying a level set approach has been performed independently by Caselles et al [7].

The outline of this paper is as follows. In Section II, we briefly explain the level set approach to front propagation problems and the accompanying numerical algorithms. In Sections III and IV, we discuss the application of this technique to shape recovery problems, and consider various speed functions and approaches to the problem, such as the effect of global speed laws, narrow band formulations, reinitialization and stopping criteria. In Section V, we present some experimental results of applying our method to some synthetic and low contrast medical images. We conclude in Section VI.

II. Front Propagation Problem

In this section we present the level set technique due to Osher and Sethian [23]. For details and an expository review, see Sethian [29].

As a starting point and motivation for the level set approach, consider a closed curve moving in the plane, that is, let $\gamma(0)$ be a smooth, closed initial curve in Euclidean plane $\Re^2$, and let $\gamma(t)$ be the one-parameter family of curves generated by moving $\gamma(0)$ along its normal vector field with speed $F(K)$, a given scalar function of the curvature K. Let $\mathbf{x}(s, t)$, be the position vector which parameterizes $\gamma(t)$ by s, $0 \le s \le S$.

One numerical approach to this problem is to take the above Lagrangian description of the problem, produce equations of motion for the position vector $\mathbf{x}(s, t)$, and then discretize the parameterization with a set of discrete marker particles lying on the moving front. These discrete markers are updated in time by approximating the spatial derivatives in the equations of motion, and advancing their positions. However, there are several problems with this approach, as discussed in Sethian [28]. First, small errors in the computed particle positions are tremendously amplified by the curvature term, and calculations are prone to instability unless an extremely small time step is employed. Second, in the absence of a smoothing curvature (viscous) term, singularities develop in the propagating front, and an entropy condition must be observed to extract the correct weak solution. Third, topological changes are difficult to manage as the evolving interface breaks and merges. And fourth, significant bookkeeping problems occur in the extension of this technique to three dimensions.

As an alternative, the central idea in the level set approach of Osher and Sethian [23] is to represent the front $\gamma(t)$ as the level set $\{\psi = 0\}$ of a function ψ. Thus, given a moving closed hypersurface $\gamma(t)$, that is, $\gamma(t = 0) : [0, \infty) \rightarrow \Re^N$, we wish to produce an Eulerian formulation for the motion of the hypersurface propagating along its normal direction with speed F, where F can be a function of various arguments, including the curvature, normal direction, etc. The main idea is to embed this propagating interface as the zero level set of a higher dimensional function ψ. Let $\psi(\mathbf{x}, t = 0)$, where $\mathbf{x} \in \Re^N$ is defined by

$$\psi(x, t = 0) = \pm d \qquad (1)$$

where d is the distance from $\mathbf{x}$ to $\gamma(t = 0)$, and the plus (minus) sign is chosen if the point $\mathbf{x}$ is outside (inside) the initial hypersurface $\gamma(t=0)$. Thus, we have an initial function $\psi(\mathbf{x}, t = 0) : \Re^N \rightarrow \Re$ with the property that

$$\gamma(t = 0) = (\mathbf{x} \mid \psi(\mathbf{x}, t = 0) = 0) \qquad (2)$$

As illustration, consider the example of an expanding circle. Suppose the initial front γ at $t = 0$ is a circle in the xy-plane (Fig. 2(a)). We imagine that the circle is the level set $\{\psi = 0\}$ of an initial surface $z = \psi(x, y, t = 0)$ in $\Re^3$ (see Fig. 2(b)). We can then match the one-parameter family of moving curves $\gamma(t)$ with a one-parameter family of moving surfaces in such a way that the level set $\{\psi = 0\}$ always yields the moving front (see Fig. 2(c) and Fig. 2(d)).

Our goal is to now produce an equation for the evolving function $\psi(\mathbf{x}, t)$ which contains the embedded motion of $\gamma(t)$ as the level set $\{\psi = 0\}$. Here, we follow the derivation presented

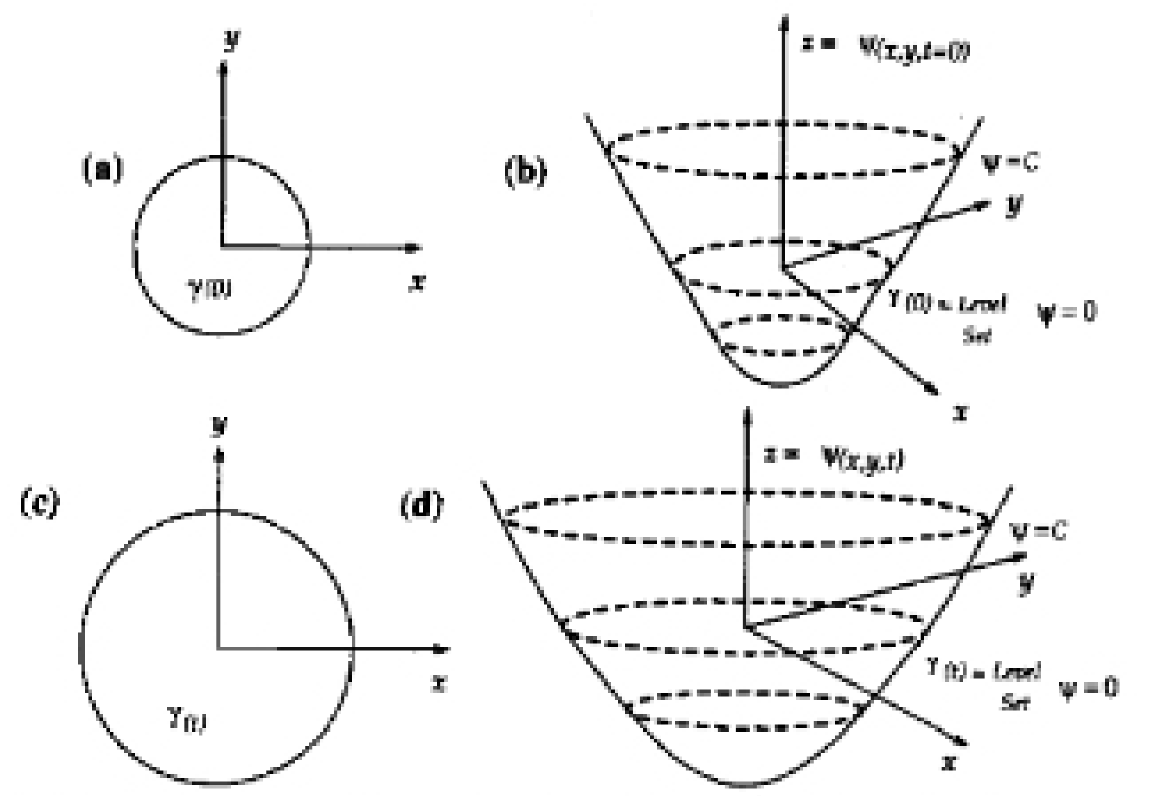

Fig. 2. Level set formulation of equations of motion – (a) and (b) show the curve γ and the surface $\psi(x, y)$ at $t = 0$, and (c) and (d) show the curve γ and the corresponding surface $\psi(x, y)$ at time t.

in [22]. Let $\mathbf{x}(t)$, $t \in [0, \infty)$ be the path of a point on the propagating front. That is, $\mathbf{x}$ $(t=0)$ is a point on the initial front $\gamma(t = 0)$, and $\mathbf{x}_t = F(\mathbf{x}(t))$ with the vector $\mathbf{x}_t$ normal to the front at $\mathbf{x}(t)$. Since the evolving function ψ is always zero on the propagating hypersurface, we must have

$$\psi(\mathbf{x}(t), t) = 0. \qquad (3)$$

By the chain rule,

$$\psi_t + \sum_{i=1}^{N} \psi_{x_i} x_{i_t} = 0 \qquad (4)$$

where x_i is the ith component of $\mathbf{x}$. Let

$$(u_1, u_2, \ldots u_N) = (x_{1_t}, x_{2_t}, \ldots, x_{N_t}).$$

Since

$$\sum_{i=1}^{N} \psi_{x_i} x_{i_t} = (\psi_{x_1}, \psi_{x_2}, \ldots, \psi_{x_N}) \cdot (u_1, u_2, \ldots, u_N) = F(\mathbf{x}(t))|\nabla\psi| \qquad (5)$$

we then have the evolution equation for ψ, namely

$$\psi_t + F|\nabla\psi| = 0 \qquad (6)$$

with a given value of $\psi(\mathbf{x}, t = 0)$. We refer to this as a Hamilton-Jacobi "type" equation because, for certain forms of the speed function F, we obtain the standard Hamilton-Jacobi equation.

There are four major advantages to this Eulerian Hamilton-Jacobi formulation. The first is that the evolving function $\psi(\mathbf{x}, t)$ always remains a function as long as F is smooth. However, the level surface $\{\psi = 0\}$, and hence the propagating hypersurface $\gamma(t)$ may change topology, break, merge, and