

**I The Domain of a Function**

Remember that the domain is the set of  $x$ 's in a function, or the set of 'first things'. For many functions, such as  $f(x) = 2x - 3$ ,  $x$  could be replaced with any real number to find  $f(x)$ . Therefore the domain could be written in set-builder notation as  $\{x \mid x \text{ is a real number}\}$  or  $(-\infty, \infty)$  using interval notation. However, not all functions have domains that are all real numbers. In real life examples, the domain is often **limited** to numbers that only make sense. For example, if  $d = 50t$  (distance in miles equals 50 times time in hours), the time can only be 0 or positive numbers. No one would travel negative hours. So the domain would be  $\{t \mid t \geq 0\}$  or  $[0, \infty)$ .

Examine the following two functions.

$$f(x) = \sqrt{4 - x} \qquad g(x) = \frac{2}{x - 3}$$

If  $x$  is replaced with a 5 in function  $f$ , the result would be  $\sqrt{-1}$ . While we've discussed imaginary numbers, when speaking of domains, we only think of real numbers. So we know 5 is not in the domain of  $f$ . How do we find the domain? We know that we can only take the square root of a positive number or zero. Therefore  $4 - x$  must be greater than or equal to zero.

$$\begin{array}{l} 4 - x \geq 0 \\ -x \geq -4 \\ x \leq 4 \end{array} \qquad \text{The domain of } f \text{ is } \{x \mid x \leq 4\} \text{ or } (-\infty, 4].$$

In function  $g$ , if  $x$  is replaced with 3, the result would be  $\frac{2}{0}$ , which is an undefined number. 3 is not in the domain of function  $g$ . We can write the domain as  $\{x \mid x \text{ is a real number and } x \neq 3\}$  or  $(-\infty, 3) \cup (3, \infty)$ .

The above examples illustrate the 'problems' you will encounter possibly when determining domain of functions. Otherwise the domain of functions will probably be all real numbers.

1. **If the function has a square root, set the radicand greater than or equal to zero and solve. The result is the domain.**
2. **If the function has a denominator, set the denominator to zero and solve. The number(s) that result(s) are excluded from the domain.**

**Ex 1:** Find the domain of each function. Write each using set-builder notation and interval notation.

a)  $f(x) = 3x^2 + 2x - 3$

b)  $g(x) = \sqrt{2x + 3}$

c)  $h(x) = \frac{3x}{x^2 - 5x + 6}$

## II The Algebra of Functions

Functions can be combined using addition, subtraction, multiplication, or division on the right sides of two given functions. The domain of the function that results will be all real numbers common to both original domains and, in case of  $\frac{f(x)}{g(x)}$ , numbers that do not make a zero denominator.

Here are the definitions of these functions given two functions  $f(x)$  and  $g(x)$ .

1. Sum Function:  $(f + g)(x) = f(x) + g(x)$
2. Difference Function:  $(f - g)(x) = f(x) - g(x)$
3. Product Function:  $(fg)(x) = f(x) \cdot g(x)$
4. Quotient Function:  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Basically, you are adding, subtracting, multiplying, or dividing the function values or  $y$  values.

**Ex 2:** Given:  $f(x) = 2x^2 - 4x + 3$  and  $g(x) = 3x - 4$ , find the following functions. State the domain of each.

a)  $(f + g)(x) =$

b)  $(g - f)(x) =$

c)  $\left(\frac{f}{g}\right)(x) =$

d)  $(gf)(x) =$

**Ex 3:** Given:  $f(x) = 3x + 2$  and  $g(x) = \sqrt{x+2}$ , find the following function values.

a)  $(f + g)(2) =$

b)  $(f \cdot g)(8) =$

c)  $\left(\frac{g}{f}\right)(10) =$

d)  $\left(\frac{f}{g}\right)(2) =$

**Note:** In the case of the quotient function, determine the domain **before** trying to simplify the quotient.

$$f(x) = x^2 - 4 \quad g(x) = x + 2$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 4}{x + 2} = \frac{(x + 2)(x - 2)}{x + 2} = x - 2$$

While the domain of expression  $x - 2$  would be all reals, you must consider there was a denominator and  $x \neq -2$ . The domain of the quotient function is  $(-\infty, -2) \cup (-2, \infty)$ .

**Ex 4:** Use the following graph to find  $(f + g)(-3)$ ,  $(fg)(5)$ , and  $(f - g)(2)$ .

