Lecture 6: Frequency Response Inspection Analysis

- Announcements:
- This is our make-up lecture
- We will have lecture tomorrow (Tuesday), as well, at our regular time and place
- Lecture Topics:
  - Amplifier Bode plot
  - Open Circuit Time Constant (OCTC) Analysis
  - Frequency Response Inspection Analysis
  - Frequency Response Examples
- --
- Last Time:

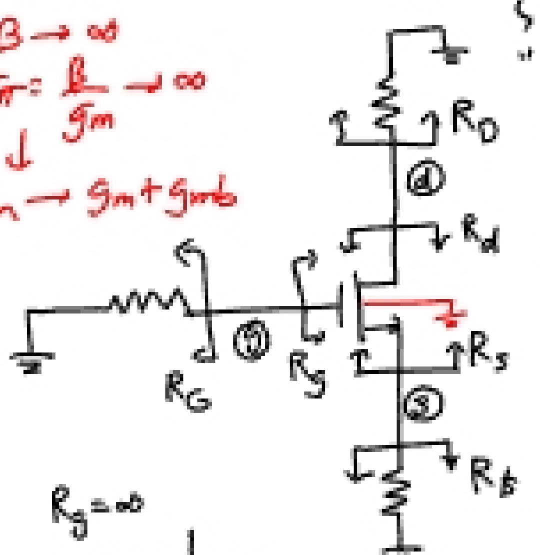
as well as tied to V_{DD}

$\boxed{\text{MOS Inspection Formulas w/ Substrate Grounded}}$

$\beta \to \infty$

$r_\pi = \frac{\beta}{g_m} \to \infty$

$g_m \to g_m + g_{mb}$



only difference from "substrate tied to source" case is that g_m is replaced by g_m+g_{mb} in some of the formulas

particularly ones where the source is involved!

$R_g = \infty$

$R_s = \frac{1}{g_m + g_{mb}}$

$R_d = r_o\left[1 + (g_m + g_{mb})R_E\right]$

over

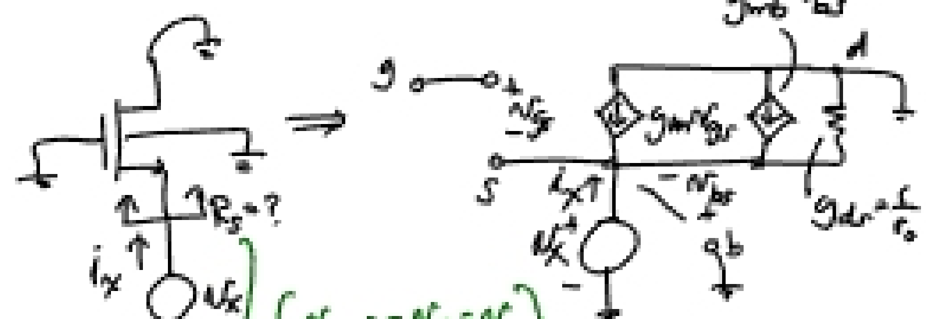
$\frac{v_d}{v_g} = -G_m R_{Ⓓ}$, $G_m = \frac{g_m}{1 + (g_m + g_{mb})R_E}$

$\frac{v_d}{v_s} = -G_m R_{Ⓓ}$, $G_m = -(g_m + g_{mb})$

$\frac{v_s}{v_g} = \frac{g_m R_E}{1 + (g_m + g_{mb})R_E}$

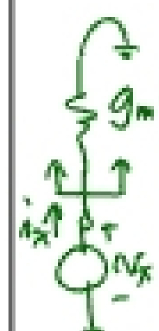
Remark: When the substrate is tied to the source, $g_{mb} = 0$.

$\boxed{\text{Effect of } g_{mb}}$ (one more example)



$v_{gs} = -v_x = v_{bs}$

$v_{ds} = -v_x$

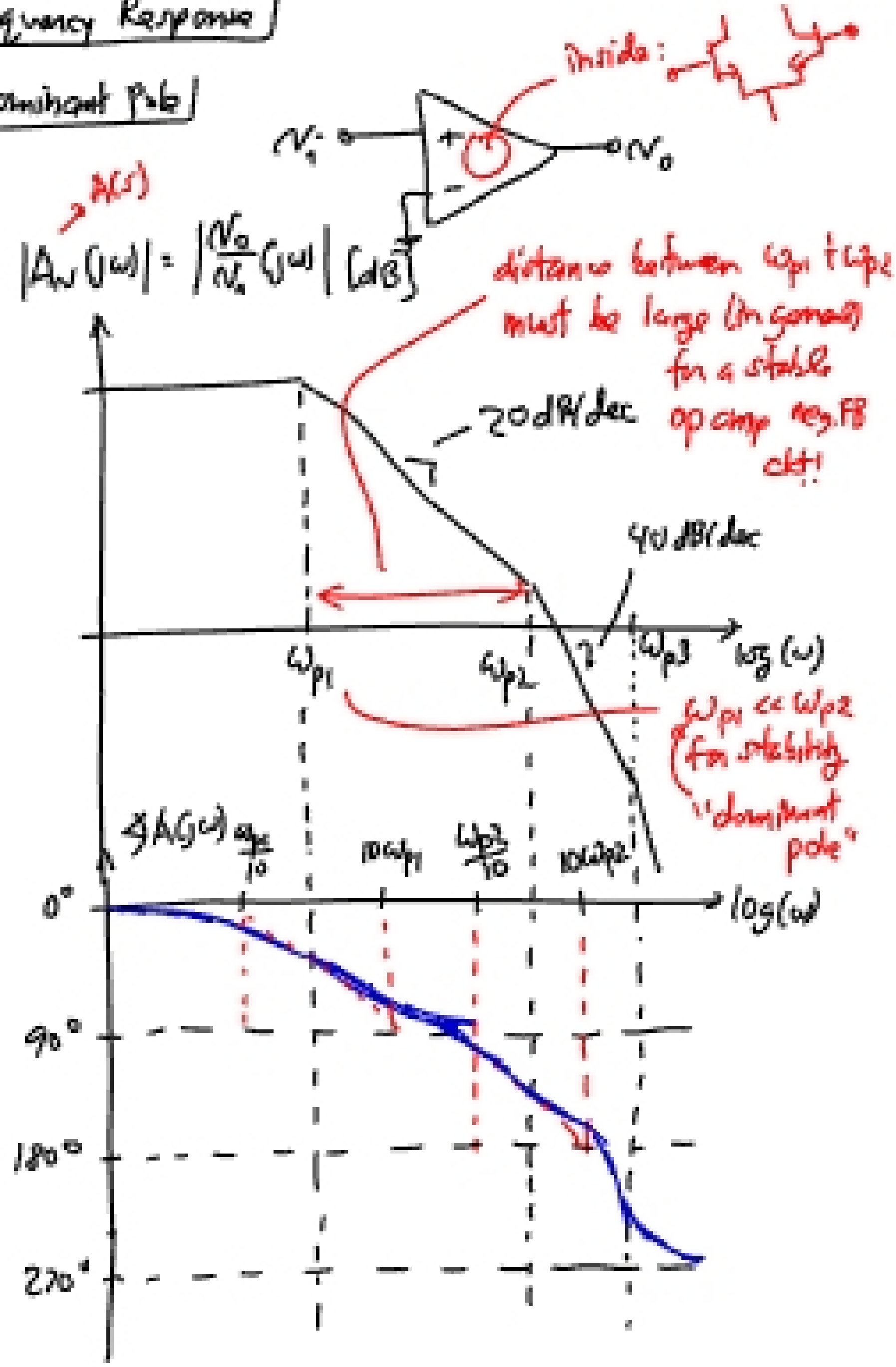


$\rightarrow R_s = \frac{1}{g_m + g_{mb} + g_{ds}} = \frac{1}{g_m} \| \frac{1}{g_{mb}} \| r_o$

$\boxed{R_s \approx \frac{1}{g_m} \| \frac{1}{g_{mb}} = \frac{1}{g_m + g_{mb}}}$

Frequency Response

Dominant Pole

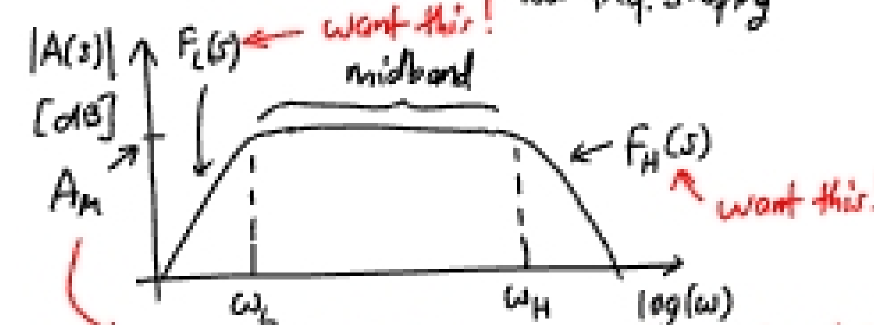


Freq. Response

Recall that the transfer function of a general amplifier can be expressed as a function of frequency via:

$$A(s) = A_M F_L(s) F_H(s)$$

midband gain; low freq. shaping; high frequency shaping



we've already found this using small-signal analysis

High Freq. Response Determination Using Open Ckt. Time Constant (OCTC) Analysis

In general:

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_{n_z} s^{n_z}}{1 + b_1 s + b_2 s^2 + \cdots + b_{n_p} s^{n_p}}, \quad n_p > n_z$$

poles, # zeros

$$= \frac{\prod_{j=1}^{n_z}\left(1 - \frac{s}{z_j}\right)}{\prod_{i=1}^{n_p}\left(1 - \frac{s}{p_i}\right)} = \frac{\prod_{j=1}^{n_z}\left(1 + \frac{s}{\omega_{zj}}\right)}{\prod_{i=1}^{n_p}\left(1 + \frac{s}{\omega_{pi}}\right)}$$

from which:

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots + \frac{1}{\omega_{pn_p}} = \sum_{i=1}^{n_p} \frac{1}{\omega_{pi}} = \sum_{k=1}^{n_p} \tau_{pk}$$

time const.

coeff. of the 1st order term

Through network theory, one can prove that: (see Gray & Meyer, Chpt. 7)

$$\sum_{i=1}^{n_p} \tau_{pi} = \sum_j C_j R_{jo} = \sum_j \tau_{jo}$$

where C_j are capacitors in the H.F. ckt., i.e., small ones
$R_{jo} \triangleq$ driving pt. resistance seen between the terminals of C_j determined with

1. all small (< 1nF) capacitors open-circuited
2. all independent sources eliminated (i.e., short voltage sources, open current sources)
3. short all large (coupling/bypass) capacitors (i.e., > 1μF or > 1nF)

In calculating the H.F. response, we use the dominant pole approximation:

(i) $\omega_{p1} \ll \omega_{p2}, \ldots, \omega_{pn_p}$ ← # poles

(ii) $F_H(s) \cong \dfrac{1}{1+\frac{s}{\omega_H}}$

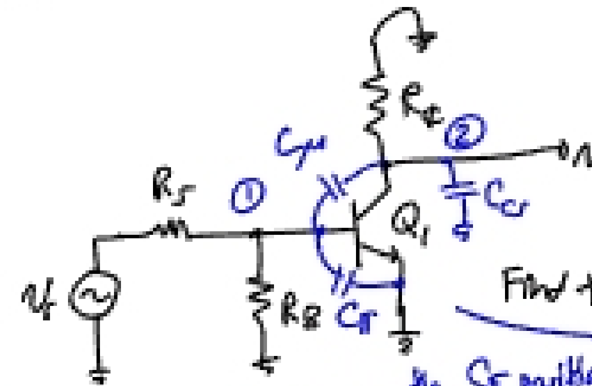
(iii) $b_1 \cong \dfrac{1}{\omega_{p1}} \rightarrow \boxed{\omega_H = \omega_{p1} \cong \dfrac{1}{b_1} = \dfrac{1}{\sum_j \tau_{jo}} = \dfrac{1}{\sum_j C_j R_{jo}}}$

When there is no dominant pole, an approximate expression for ω_H is:

$$\omega_H \approx \frac{1}{\sqrt{\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + \cdots - \frac{1}{\omega_{z1}^2} - \frac{1}{\omega_{z2}^2} - \cdots}}$$

(just FYI)

Example. H.F. Analysis of a C.E. Ckt.

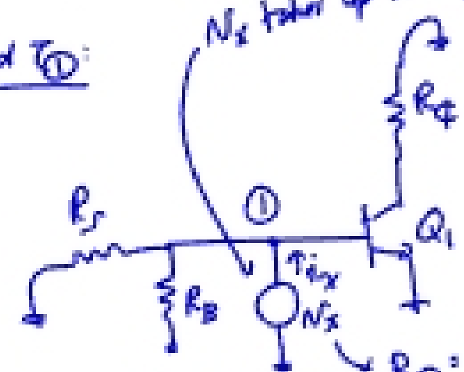


"3dB freq."
Find the cut-off freq., ω_H.

< 1nF caps: $C_\pi, C_\mu, \& C_L$ all open-ckted

Find $\tau_{①}$:

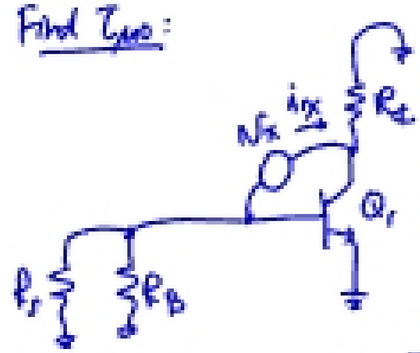
v_x takes up the C_π position.



$R_{①} = \dfrac{v_x}{i_x} = r_\pi \| R_S \| R_B$

$C_{①} = C_\pi \longrightarrow \tau_{①} = C_\pi (r_\pi \| R_S \| R_B)$

Find $\tau_{\mu o}$:



"O" means "in-shunt"

$R_{\mu o} = \dfrac{v_x}{i_x} = R_{①} + R_{②} + g_m R_{①} R_{②}$

$R_{\mu o} = R_S \| R_B \| r_\pi + r_o \| R_C + g_m (R_S \| R_B \| r_\pi)(r_o \| R_C)$

$\tau_{\mu o} = R_{\mu o} C_\mu$

$G_m = \dfrac{g_m}{1 + g_m R_E} = g_m$ (R_E = 0)