Graphs

- Formal Definition of Graph G is 2 finite sets
  - V(G) = set of vertices & E(G) = set of edges
- Example:
  - V(H) = {a,b,c,d,e}   E(H) = {{a,c},{c,e},{e,b},{b,d},{d,a}}
  - V(K) = {a,b,c,d} E(K)= {(a,b),(b,a),(a,d),(d,a),(c,c)}
- Variations
  - digraph : edges are ordered tuples
  - multi-graph : edge list is a mulitset (bag) not set
  - simple graph : no parallel edges and no "reflexive loops"
  - connected graph: can get from any vertex to any other
  - complete graph: has an edge for every pair of vertices
  - complete bipartite graph: 2 subsets of vertices (u and v), edge from each v to each u, no edges connecting u elements and no edges connecting v elements
- Subgraph: H is a subgraph of G $\leftrightarrow$ V(H) $\subseteq$V(G) and E(H)$\subseteq$E(G)

Counting in Graphs

- Number of Edges Possible
  - complete (simple) graph $n(E(K_x)) = \sum_{i=1}^{x-1} i$
  - complete bipartite graph $n(E(K_{x,y})) = x * y$
- Degree of a Vertex

  = number of times that vertex is the endpoint of an edge

  = number of edges incident on it with self-loops counted twice

Isomorphism

- (G is isomorphic to H) $\leftrightarrow$ There exists a bijective function f_1:(V(G)) $\rightarrow$ V(H) and a bijective function f_2:(E(G)) $\rightarrow$ E(H)

Traversing a Graph

Name	Repeated Edges	Repeated Verticies	Same end/start
Walk	allowed	allowed	allowed
Path	NO	allowed	allowed
Simple Path	NO	NO	NO
Closed Walk	allowed	allowed	YES
Circuit	NO	allowed	YES
Simple Circuit	NO	only the start/end	YES

Euler Circuit

- A circuit that contains every edge and every vertex
  - starts and stops at the same point
  - uses every vertex at least once
  - uses every edge exactly once
- G has an Euler Circuit $\leftrightarrow$ G is a connected graph and
  Every vertex of G has even degree

Hamiltonian Circuit

- A simple circuit that contains every vertex.
  - starts and stops at the same point
  - uses every vertex exactly once (except the first and last)
  - does not repeat an edge