16.07 Lab II

Issued: Monday, November 2, 2009

Due: Monday, November 23, 2009

Introduction

In this second numerical laboratory exercise, you will build upon the simulation developed in the first lab. Here, you will use your capability to change from the Earth-fixed inertial reference frame, to the Earth-Moon rotating reference frames, in which the Earth and Moon are fixed in the frame of reference. This frame change will help you explore different orbital phenomena within the framework of the restricted three-body problem, such as free-return trajectories to the moon, the behavior of orbits in the Earth-Moon neighbor as well as the behavior of spacecraft near the Lagrange points, both stable (L4, L5) and unstable (L1, L2). In the restricted three-body problem, we consider the motion of three bodies, but do not consider the gravitational force from the smaller body, the spacecraft, on the two large bodies, called the primaries.

In lab 1, you used the radius of the earth and your length scale, and 1 day (24 hours) as your time scale. This results in the following values for the important parameters of the system.

Table 1: Non-Dimensional Physical and Orbital Data

μ_{earth}	11468
μ_{moon}	141
R-distance between moon and earth	60.269
r_0-distance between earth and origin	.7324
r_1-distance between moon and origin	59.53
Ω-rotation rate of coordinate system	.230325

If you were successful in getting the earth and the moon to remain fixed in your rotating coordinate system by adjusting slightly the value of Ω, you may place the earth and moon at these fixed points and only consider the motion of the spacecraft in the gravitational fields of the earth and moon in the rotating coordinate system using this value of Ω. (Since we are going to ignore the effects of the spacecraft on the earth and moon, we leave the earth and moon at these fixed positions.) If you did not obtain fixed positions for the earth and the moon in the rotating coordinate system, get cracking. I don't think Lab 2 will behave without this.

Problems

In the following problems, the items you must turn in—either the written answers to questions, or required plots—are highlighted in **bold.** The ordering of these deliverables in your report should match the ordering that appears below.

1.Earth Fixed Satellites

In this problem, we revisit the restricted three-body problem simulation from Lab I. Our first task is to place a satellite in a circular orbit at a radius R from the center of the earth of magnitude $R = 2R_{earth}$. The proper choice of boundary conditions for this calculation is subtle. As shown in the figure, in an inertial coordinate system, placing a satellite in earth orbit requires matching the boundary condition with the rotation of the earth-moon system about its center of mass. Consider for simplicity only the insertion point directly beyond the earth, on the line joining the earth-moon centers, the line of symmetry.

For a counterclockwise insertion, we would require a negative y velocity of magnitude relative to a fixed earth in an inertial frame.

$$v_{s_i} = -\sqrt{\mu/r_s} \tag{1}$$

Therefore, when we move into the rotating coordinate system in which the earth and moon are fixed in our coordinate system, we obtain the following boundary conditions:

$$v_s = \pm\sqrt{\mu/r_s} + \Omega * r_s \tag{2}$$

where for a counterclockwise orbit with insertion point on the far side of the earth, the initial y velocity is negative; for a clockwise rotation, the y velocity is positive. Given the rotation of the earth, the counterclockwise rotation would be preferred.

Your mission is to place a satellite in earth orbit at twice earth radius using the insertion point on the line of symmetry at the far side of the earth from the moon. **Do both the clockwise and counterclockwise rotation. Verify that the orbits are circular to the accuracy of your calculations.** Note that the time required to complete an orbit is slightly different for each direction. This has to do with the rotation of the coordinate system.

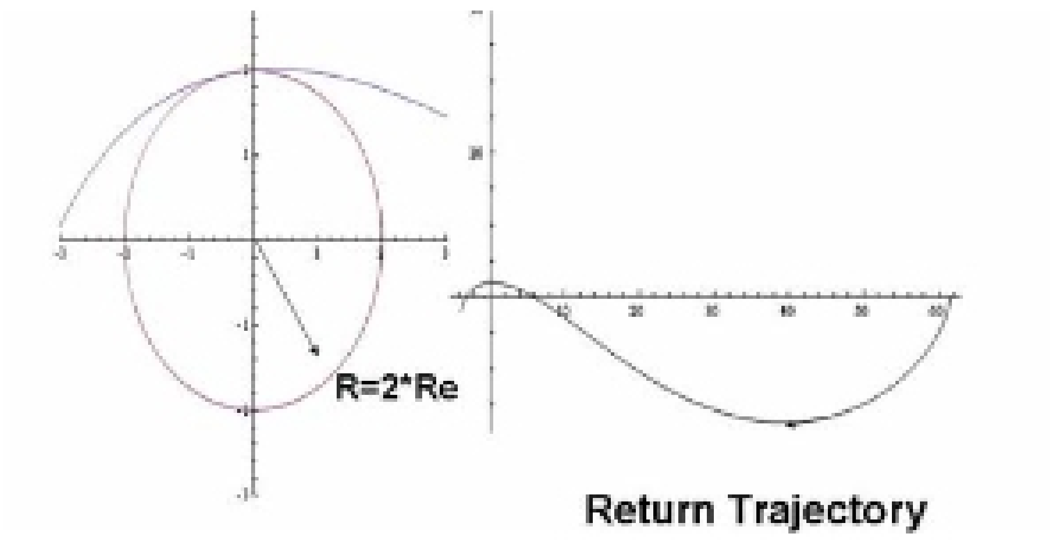
2A. Off to the Moon: First Attempt

Now beginning with the insertion of the satellite in a counterclockwise obit at twice the radius of the earth from the back-side of the earth on the line of symmetry. Begin to increase the magnitude of the insertion velocity above the magnitude required for a circular orbit. It is useful to scale the velocity as $v_s = A\sqrt{\mu/r_s}$ and focus on determining A for interesting motions. For example, what is A to "escape" earth's orbit; this is clearly an upper limit on A, modified only slightly by the presence of the moon.

Run several cases from $A = 1$ to $A < A_{escape}$ (Try A=1.36, 1.37, 1.38 and 1.39). What happens? Now insert the spacecraft at $y3(0) = -2$ and $x3(0) = x1$ where $x1$ is the position of the earth. Repeat your calculations for $A = 1.39$. **Present 4 plots of your most interesting results. Interpret your results.** Do you think you can get to the moon this way?

2B. Off to the Moon: Second Attempt

The best way to get to the moon, is to go there and try to get back to the earth. Then use the image theorem and flip the orbit to get back to the moon—like the astronauts tossing cheeseburgers, remember how well that worked. Place a satellite in a planar x, y circular *clockwise* orbit around the moon at a radius of r_{sm}. The point of insertion is on the far side of the moon on the line of symmetry joining the earth-moon. The choice of radius is up to you, but I had good luck with $r_{sm} = 1.4R_{earth}$, $r_{sm} = R_{earth}$ and $r_{sm} = 2 * R_{earth}$ with the mission ending up in a parking orbit around the earth at $R = 2R_{earth}$, and $R = 1.5R_{earth}$.
Verify that a circular orbit results from your choice of initial condition.
Now, defining the velocity for a circular orbit of radius r_{sm} around the moon as v_{moon}, increase the velocity as $v = \mathbf{A} * v_{moon}$, for different values of **A**. The notion of escape velocity does not quite exist for the moon; you can escape the moon but you will be back in the influence of the earth–which is of course where you want to be.
Now, the goal is to increase **A** until the resulting orbit passes close enough to the earth to be inserted into a parking orbit. You should be able to get the orbit from moon to earth at $R = 2 * R_{earth}$ and/or $R = 1.5R_{earth}$. Run several cases and determine **A** which makes this possible. **Determine exactly what $x(t)$, $y(t)$, $x'(t)$, $y'(t)$ are for the conditions at the point at which the orbit is tangent to the earth parking orbit at 2 earth radii from the center of the earth. Also, at what elapsed time does the spacecraft arrive at this point on its return? Determine exactly what $x(t)$, $y(t)$, $x'(t)$, $y'(t)$ are for the point at which the orbit is 1.5 earth radii from the center of the earth. Also, at what elapsed time does the spacecraft arrive at this point?** Recall that you are using 1 day as your time scale. Therefore, the time obtained is the time in days required to "return" from the moon on a free-return trajectory.



Return Trajectory