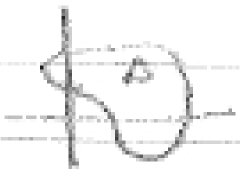
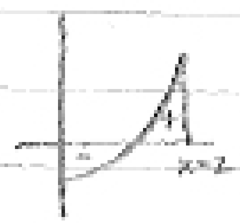
10/20/14

Double integral



$\iint_A f(p)\, dA$

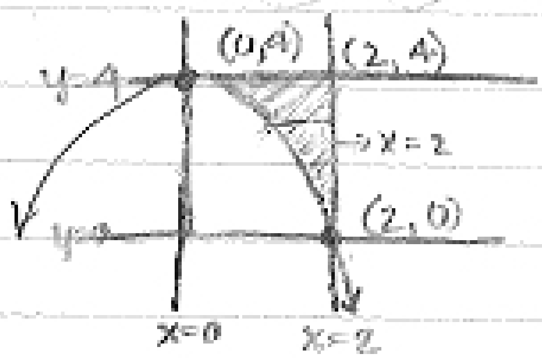


$y = x^2 - 1$

$\int_0^2 x^2 - 1$

$= \frac{1}{3}x^3 - x \Big|_0^2 = \boxed{2/3}$

$\int_0^2 \int_{4-x^2}^4 dy\,dx$ ← switch



$x = 2 \quad y = 4$

$x = 0 \quad y = 4 - x^2$

$\int \int dx\,dy$

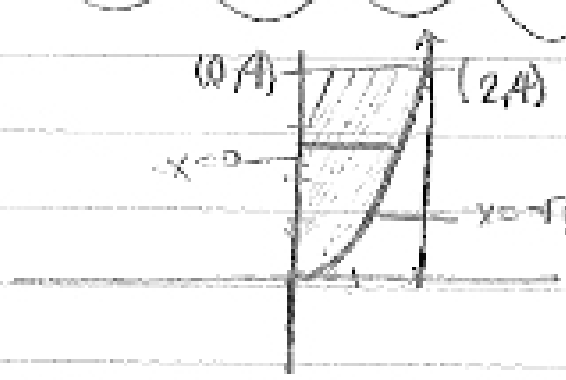
$x^2 = 4y$

$x = \pm\sqrt{4-y}$

$x = +\sqrt{4-y}$

$\boxed{\int_0^4 \int_{\sqrt{4-y}}^2 dx\,dy}$ ← equals the original dydx integral.

$\int_0^2 \int_{x^2}^4 dy\,dx$



$x = 2 \quad y = 4$

$x = 0 \quad y = x^2$

$\boxed{\int_0^4 \int_0^{\sqrt{y}} dx\,dy}$

$x = \pm\sqrt{y} \Rightarrow \sqrt{y}$

Actually integrate problem from previous page

$\int_0^2 \int_{4-x^2}^4 1 \, dy\,dx$

$\int_0^2 y \Big|_{4-x^2}^4 dx$

$\int_0^2 4-(4-x^2)\,dx = \int_0^2 x^2\,dx = \frac{1}{3}x^3 \Big|_0^2 = \boxed{8/3}$

$\int_0^2 \int_0^2 9-x^2-y^2 \, dx\,dy$

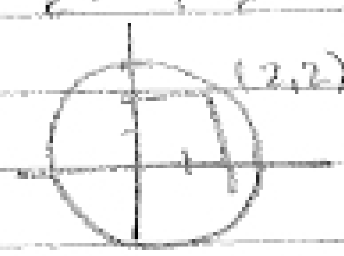
$\int_0^2 9x - \frac{1}{3}x^3 - xy^2 \Big|_{x=0}^2 dy$

$\int_0^2 [(18 - 8/3 - 2y^2) - (0-0-0)]\,dy = \int_0^2 \frac{46}{3} - 2y^2\,dy$

$= \frac{46}{3}y - \frac{2}{3}y^3 \Big|_0^2$

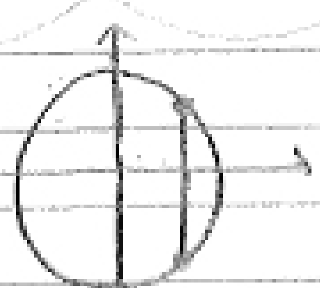
$= \left(\frac{92}{3} - \frac{16}{3}\right) - 0 = \boxed{\frac{76}{3}}$

$z = 9 - x^2 - y^2$



$z = 9 - (2)^2 - (2)^2$ = pos.

thus, inside.



$\iint_A 9-x^2-y^2 \, dA$ $\qquad y = \pm\sqrt{9-x^2}$

$x^2+y^2 \le 9$ $\qquad \iint 9-x^2-y^2 \, dy\,dx$

$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{+\sqrt{9-x^2}} 9-x^2-y^2 \, dy\,dx$

$\int_{-3}^{3} 9y - x^2y - \frac{1}{3}y^3 \Big|_{y=-\sqrt{9-x^2}}^{\sqrt{9-x^2}} dx$

$\left(9(\sqrt{9-x^2}) - x^2(\sqrt{9-x^2}) - \frac{(9-x^2)^{3/2}}{3}\right) - \left(9(\sqrt{9-x^2}) - x^2(-\sqrt{\ldots}\right)$

$= 2\int_{-3}^{3} \left[9-x^2 - \frac{1}{3}(9-x^2)\right]\sqrt{9-x^2}\,dx$

next page

$$= 2 \int_{-3}^{3} \sqrt{9-x^2}\,\left(\tfrac{2}{3}\right)(9-x^2)\,dx$$

$$= \tfrac{4}{3} \int_{-3}^{3} (9-x^2)^{3/2}\,dx$$

$\cancel{u = 9 - x^2}$

$\cancel{du = -2x\,dx}$

$$= \tfrac{4}{\cancel{3}} \int_{-\pi/2}^{\pi/2} 27\cos^3\theta \cdot \cancel{3}\cos\theta\,d\theta$$

$x = 3\sin\theta$

$$(27)(4) \int_{-\pi/2}^{\pi/2} \cos^4\theta\,d\theta$$

$dx = 3\cos\theta\,d\theta$

$9 - x^2 = 9 - 9\sin^2\theta = 9(1 - \sin^2\theta)$

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

This is crazy! Switch to polar!