Astro 321

Set 7: Spherical Collapse & Halo Model

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Closed Universe

- Friedmann equation in a closed universe

$$\frac{1}{a}\frac{da}{dt} = H_0 \left(\Omega_m a^{-3} + (1-\Omega_m)a^{-2}\right)^{1/2}$$

- Parametric solution in terms of a development angle
  $\theta = H_0\eta(\Omega_m - 1)^{1/2}$, scaled conformal time η

$$\begin{aligned} r(\theta) &= A(1-\cos\theta) \\ t(\theta) &= B(\theta - \sin\theta) \end{aligned}$$

  where $A = r_0\Omega_m/2(\Omega_m - 1)$, $B = H_0^{-1}\Omega_m/2(\Omega_m - 1)^{3/2}$.

- Turn around at $\theta = \pi$, $r = 2A$, $t = B\pi$.
- Collapse at $\theta = 2\pi$, $r \to 0$, $t = 2\pi B$

Spherical Collapse

- Parametric Solution: