Sets

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Spring 2006

Computer Science & Engineering 235
Introduction to Discrete Mathematics
Sections 1.6 – 1.7 of Rosen
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Notes

Introduction I

We've already implicitly dealt with sets (integers, $\mathbb{Z}$; rationals ($\mathbb{Q}$) etc.) but here we will develop more fully the definitions, properties and operations of sets.

Definition

A *set* is an unordered collection of (unique) objects.

Sets are fundamental discrete structures that form the basis of more complex discrete structures like graphs.

Contrast this definition with the one in the book (compare *bag*, *multi-set*, *tuples*, etc).

Definition

Notes

Introduction II

The objects in a set are called *elements* or *members* of a set. A set is said to *contain* its elements.

Recall the notation: for a set A, an element x we write

$$x \in A$$

if A contains x and

$$x \notin A$$

otherwise.

Latex notation: \in, \neg\in.

Notes

Terminology I

Definition

Two sets, A and B are *equal* if they contain the same elements. In this case we write $A = B$.

Example

$\{2,3,5,7\} = \{3,2,7,5\}$ since a set is *unordered*.

Also, $\{2,3,5,7\} = \{2,2,3,3,5,7\}$ since a set contains *unique* elements.

However, $\{2,3,5,7\} \neq \{2,3\}$.

Notes

Terminology II

A multi-set is a set where you specify the number of occurrences of each element: $\{m_1 \cdot a_1, m_2 \cdot a_2, \ldots, m_r \cdot a_r\}$ is a set where m_1 occurs a_1 times, m_2 occurs a_2 times, etc.

Note in CS (Databases), we distinguish:

- a set is w/o repetition
- a bag is a set with repetition

Notes

Terminology III

We've already seen *set builder* notation:

$$O = \{x \mid (x \in \mathbb{Z}) \wedge (x = 2k \text{ for some } k \in \mathbb{Z})\}$$

should be read O is the set that contains all x such that x is an integer and x is even.

A set is defined in *intension*, when you give its set builder notation.

$$O = \{x \mid (x \in \mathbb{Z}) \wedge (x \leq 8)\}$$

A set is defined in *extension*, when you enumerate all the elements.

$$O = \{0, 2, 6, 8\}$$

Notes

Venn Diagram

Example

A set can also be represented graphically using a *Venn diagram*.

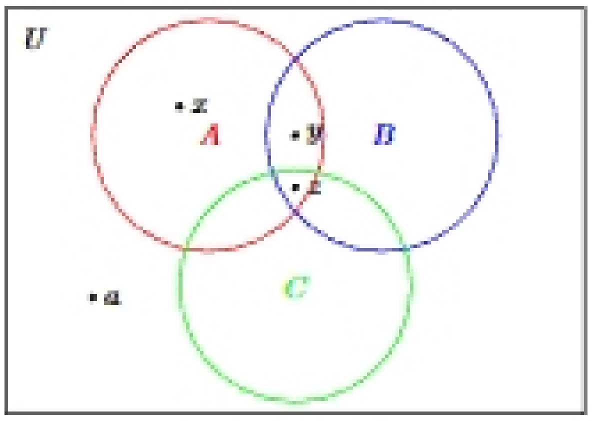

Figure: Venn Diagram

Notes

More Terminology & Notation I

A set that has no elements is referred to as the *empty set* or *null set* and is denoted $\emptyset$.

A *singleton* set is a set that has only one element. We usually write $\{a\}$. Note the different: brackets indicate that the object is a *set* while a without brackets is an *element*.

The subtle difference also exists with the empty set: that is

$$\emptyset \neq \{\emptyset\}$$

The first is a set, the second is a set containing a set.

Notes

More Terminology & Notation II

Definition

A is said to be a subset of B and we write

$$A \subseteq B$$

if and only if every element of A is also an element of B.

That is, we have an equivalence:

$$A \subseteq B \iff \forall x (x \in A \rightarrow x \in B)$$

Notes