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Chapter 5 : Transient Conduction

$T(t)$ or $T(x,t)$
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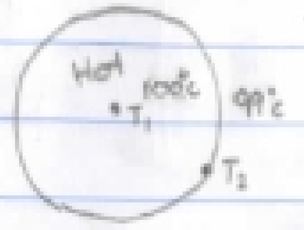
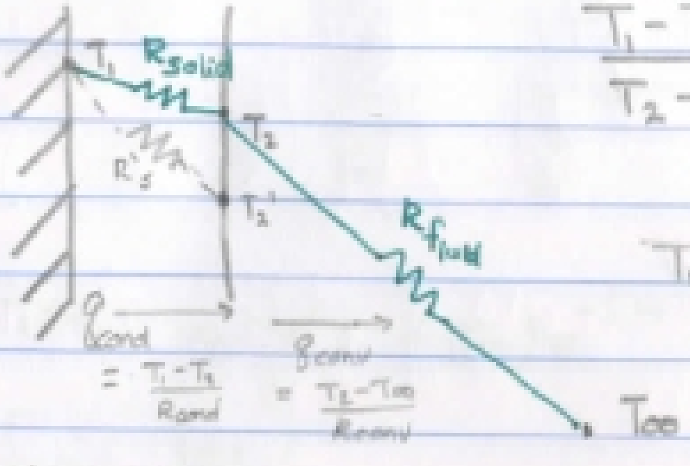


T_∞ cold = 20°c



$\frac{T_1 - T_2}{T_2 - T_\infty} << 1$ ie $< \frac{1}{10}$ or 0.1

$T_1 \approx T_2 = T$

for steady state: $q_{cond} = q_{conv}$

$\frac{T_1 - T_2}{R_{cond}} = \frac{T_2 - T_\infty}{R_{conv}} \longrightarrow \frac{T_1 - T_2}{T_2 - T_\infty} = \frac{R_{cond}}{R_{conv}} << 1$ or < 0.1

Lump Capacitance Model
L.C.M
$T(x,t) \rightarrow T(t)$

$\frac{R_{cond} = \frac{L}{kA}}{R_{conv} = \frac{1}{hA}} \Rightarrow \frac{hL_c}{k_s}$ where $L_c \equiv \frac{Vol}{A_w}$

If $Biot_{L_c} << 1 \longrightarrow$ then $T_{solid} \approx$ uniform

$\dot{E}_{st} = \cancel{\dot{E}_{in}}^{0} - \dot{E}_{out} + \cancel{\dot{E}_{gen}}^{0} \longrightarrow \dot{E}_{st} = -\dot{E}_{out}$

$\dot{E}_{out} \Rightarrow q_{conv} = hA_w(T(t) - T_\infty)$

$\dot{E}_{st} = mc\frac{dT}{dt}$

$\dot{E}_{st} = -\dot{E}_{out}$

$mc\frac{dT}{dt} = -hA(T(t) - T_\infty) \longrightarrow \frac{dT}{dt} = \frac{-hA}{mc}\underbrace{(T - T_\infty)}_{\theta}$

$\rightarrow \frac{d\theta}{dt} = \frac{-hA}{mc}\theta \rightarrow \theta_i = \theta(t=0) = 80°c \rightarrow \theta(t) = \theta_i e^{-\frac{hA}{mc}t}$

$y(t) = 80°c\, e^{-\frac{hA}{mc}t}$

$\boxed{T(t) - T_\infty = (T_i - T_\infty)e^{-\frac{hA}{mc}t}}$

$$T(t) - T_\infty = (T_i - T_\infty)\, e^{-\frac{hA}{mc}t}$$

$$-\frac{hA}{mc}t \rightarrow m = \rho V \rightarrow \frac{-hA}{\rho V c}t \rightarrow L_c = \frac{V}{A} \rightarrow \frac{-h}{\rho c L_c}t \rightarrow \frac{-hk}{\rho c k L_c} \rightarrow$$

equal

$$\rightarrow \alpha = \frac{k}{\rho c} \rightarrow \underbrace{\left(\frac{\alpha t}{L_c^2}\right)}_{\substack{\text{fourier number} \\ Fo}} \underbrace{\left(\frac{hL_c}{k_s}\right)}_{\substack{\text{Biot number} \\ Bi}}$$

(5.13) $$T(t) - T_\infty = (T_i - T_\infty)\, e^{-(Fo \cdot Bi)}$$

$$\boxed{\begin{aligned} Bi &= \frac{hL_c}{k_s} \\ Fo &= \frac{\alpha t}{L_c^2} \end{aligned}} \qquad \alpha = \frac{k}{\rho c}$$