ME451: Control Systems

Lecture 5
Modeling of mechanical systems

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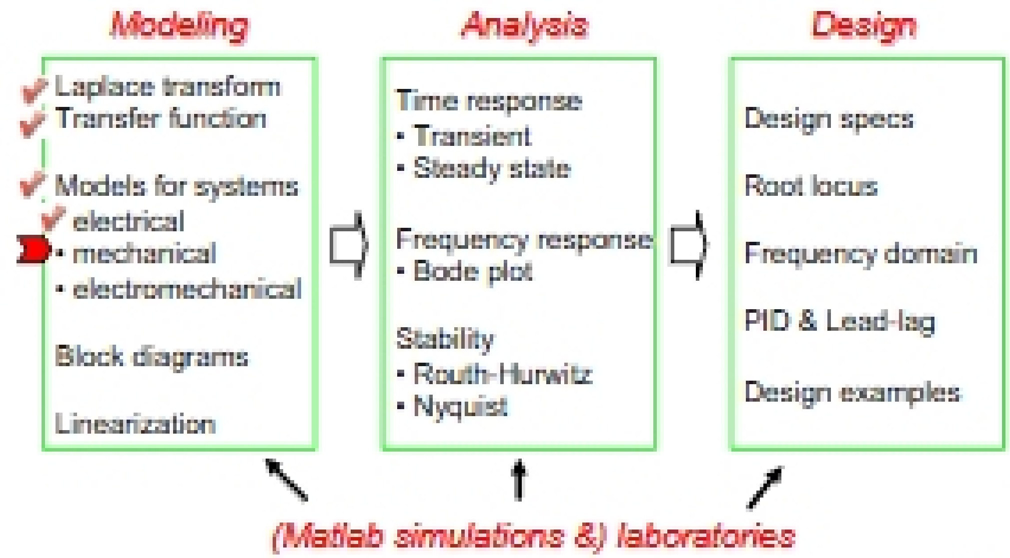
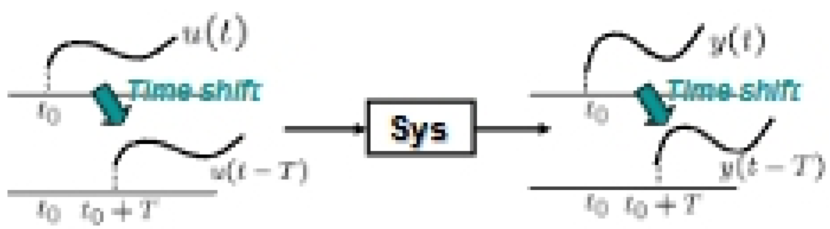
Course roadmap



Time-invariant & time-varying

- A system is called *time-invariant (time-varying)* if system parameters do not (do) change in time.
- Example: Mx''(t)=f(t) & M(t)x''(t)=f(t)
- For time-invariant systems:



- This course deals with time-invariant systems.

Newton's laws of motion

- 1st law:
  - A particle remains at rest or continues to move in a straight line with a constant velocity if there is no unbalancing force acting on it.
- 2nd law:
  - $\sum F_i(t) = m\frac{d^2x}{dt^2}$: translational
  - $\sum \tau_i(t) = I\frac{d^2\theta}{dt^2}$: rotational
- 3rd law:
  - For every action has an equal and opposite reaction

Translational mechanical elements: (constitutive equations)

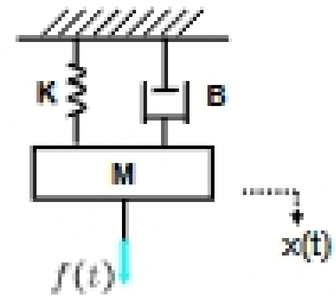
Mass | Spring | Damper

$f(t) = Mx''(t)$ $\quad$ $f(t) = K(x_1(t) - x_2(t))$ $\quad$ $f(t) = B(x_1'(t) - x_2'(t))$

$\begin{pmatrix} x(0)=0 \\ \dot{x}(0)=0 \end{pmatrix}$ $\quad$ $\begin{pmatrix} x_1(0)=0 \\ x_2(0)=0 \end{pmatrix}$

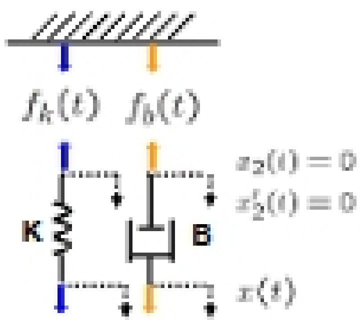
$F(s) = Ms^2X(s)$ $\quad$ $F(s) = K(X_1(s) - X_2(s))$ $\quad$ $F(s) = Bs(X_1(s) - X_2(s))$

Mass-spring-damper system



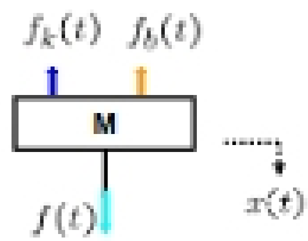
$$Mx''(t) + Bx'(t) + Kx(t) = f(t)$$

Free body diagram



$f_k(t) = K(x(t) - 0)$ $\quad$ $f_b(t) = B(x'(t) - 0)$

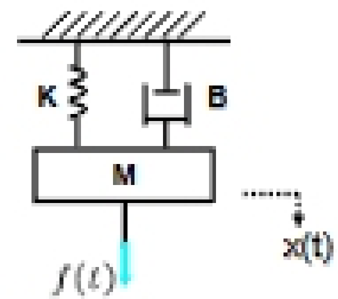
Direction of actual force will be automatically determined by the relative values!



- Newton's law: F=ma

$$Mx''(t) = f(t) - f_k(t) - f_b(t) = f(t) - Kx(t) - Bx'(t)$$

Mass-spring-damper system



- Equation of motion

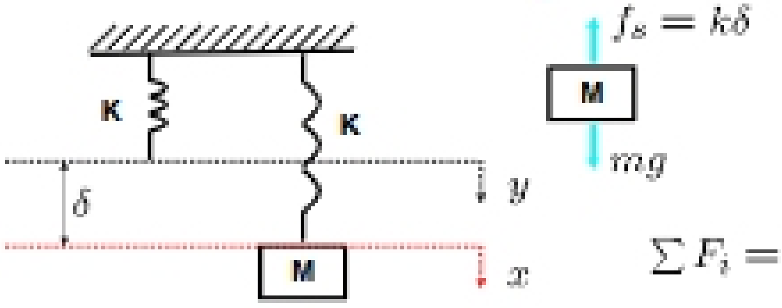
$$Mx''(t) + Bx'(t) + Kx(t) = f(t)$$

- By Laplace transform (with zero initial conditions),

$$X(s) = \frac{1}{Ms^2 + Bs + K}F(s)$$

(2nd order system)

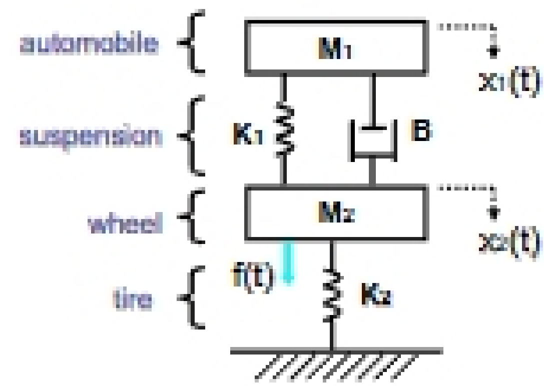
Gravity?



$$\sum F_i = ma$$

- At rest, $\sum F_i = -k\delta + mg = 0$
- y coordinate: $m\ddot{y} = mg - ky$
- x coordinate: $m\ddot{x} = mg - k(x+\delta)$
  $= -kx$

Automobile suspension system



$$\begin{cases} M_1 x_1''(t) = -B(x_1'(t) - x_2'(t)) - K_1(x_1(t) - x_2(t)) \\ M_2 x_2''(t) = f(t) - B(x_2'(t) - x_1'(t)) - K_1(x_2(t) - x_1(t)) - K_2 x_2(t) \end{cases}$$

Automobile suspension system

$$\begin{cases} M_1 x_1''(t) = -B(x_1'(t) - x_2'(t)) - K_1(x_1(t) - x_2(t)) \\ M_2 x_2''(t) = f(t) - B(x_2'(t) - x_1'(t)) - K_1(x_2(t) - x_1(t)) - K_2 x_2(t) \end{cases}$$

Laplace transform with zero ICs

$$\begin{cases} M_1 s^2 X_1(s) = -B(sX_1(s) - sX_2(s)) - K_1(X_1(s) - X_2(s)) \\ M_2 s^2 X_2(s) = F(s) - B(sX_2(s) - sX_1(s)) - K_1(X_2(s) - X_1(s)) - K_2 X_2(s) \end{cases}$$

$$\begin{cases} X_1(s) = \underbrace{\frac{Bs + K_1}{M_1 s^2 + Bs + K_1}}_{G_1(s)} X_2(s) \\ X_2(s) = \frac{1}{M_2 s^2 + Bs + K_1 + K_2} F(s) + \frac{Bs + K_1}{M_2 s^2 + Bs + K_1 + K_2} X_1(s) \end{cases}$$

Block diagram