EE 140: *Analog Integrated Circuits*
Lecture 7: High Frequency Inspection Analysis

CTN 2/10/09

Logistics:

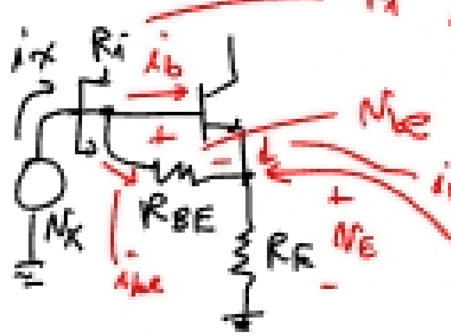
- I will miss Thursday, Feb. 12
- Make up lecture time:
  - ↳ Tonight, Tuesday, 7 p.m., 247 Cory

Today:

- High Frequency Cut-Off Examples
- Short-Circuit Time Constant Analysis for Low Frequency Cut-Off

Problem 1(c)

$R_i = \frac{v_x}{i_x}$



$i_x = i_b + i_{be}$

$$v_x = \underbrace{i_x (r_\pi \| R_{BE})}_{v_{be}} + \underbrace{\overbrace{[(\beta+1) i_b + i_x - i_b]}^{(\beta i_b + i_x) R_E} R_E}_{v_E}$$

Current Divider:

$$\left[i_b = i_x \left(\frac{R_{BE}}{r_\pi + R_{BE}} \right) \right]$$

$$v_x = i_x \left[(r_\pi \| R_{BE}) + \left[\beta \left(\frac{R_{BE}}{r_\pi + R_{BE}} \right) + 1 \right] R_E \right]$$

$$R_i = \frac{v_x}{i_x} = (r_\pi \| R_{BE}) + (\beta' + 1) R_E$$

$$\beta' = \beta \left(\frac{R_{BE}}{r_\pi + R_{BE}} \right)$$

Insp. Analysis Formula:

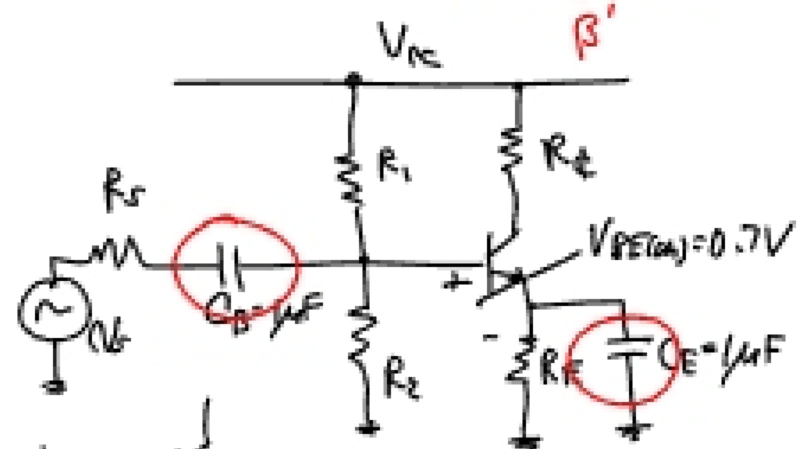
$$R_i = \underline{r_\pi + (\beta+1) R_E} \simeq \underline{r_\pi (1 + g_m R_E)}$$

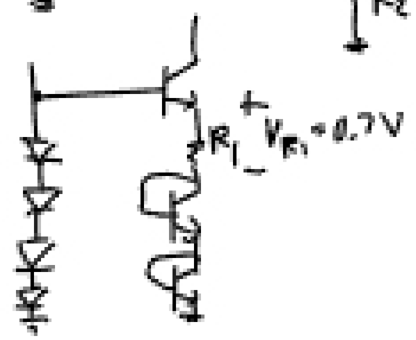
$$R_i = (r_\pi \| R_{BE})(1 + g_m R_E)$$

$$= (r_\pi \| R_{BE}) + \underbrace{g_m}_{\beta} \left(\frac{r_\pi R_{BE}}{r_\pi + R_{BE}} \right) R_E$$

$$= (r_\pi \| R_{BE}) + \underbrace{\beta \left(\frac{R_{BE}}{r_\pi + R_{BE}} \right)}_{\beta'} R_E$$

$=$ ✓





Last Time – OCTC

Find τ_1:

$R = \frac{v_x}{i_x} = r_\pi \| R_s \| R_B$

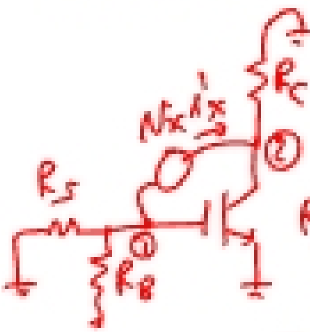
$\tau_1 = C_\pi (r_\pi \| R_s \| R_B)$

Find $\tau_{\mu o}$: replace C_μ w/ v_x

use impedance formula

"0" means "in-shunt"

$R_{\mu o} = \frac{v_x}{i_x} = R_1 + R_2 + g_m R_1 R_2$



$R_{\mu o} = R_s \| R_B \| r_\pi + r_o \| R_c + g_m (R_s \| R_B \| r_\pi)(r_o \| R_c)$

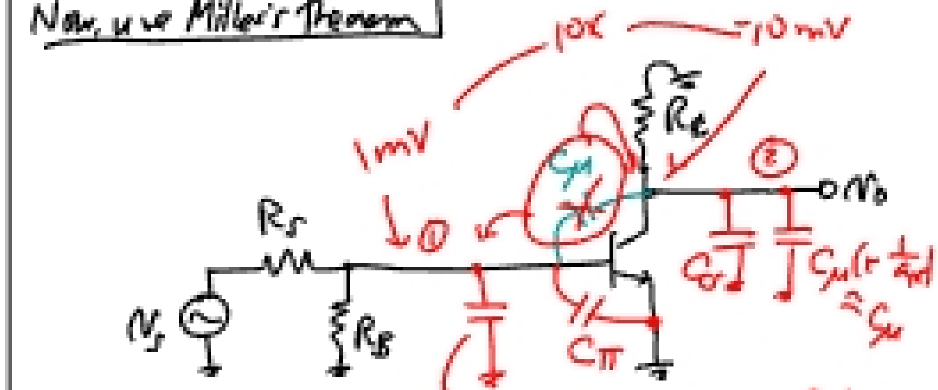
$\tau_{\mu o} = R_{\mu o} C_\mu$

Find τ_2:

$\tau_2 = C_{cs}(r_o \| R_L) \approx C_{cs} R_L$

$$\boxed{\omega_H = \frac{1}{\tau_1 + \tau_{\mu o} + \tau_2}}$$

Now, use Miller's Theorem



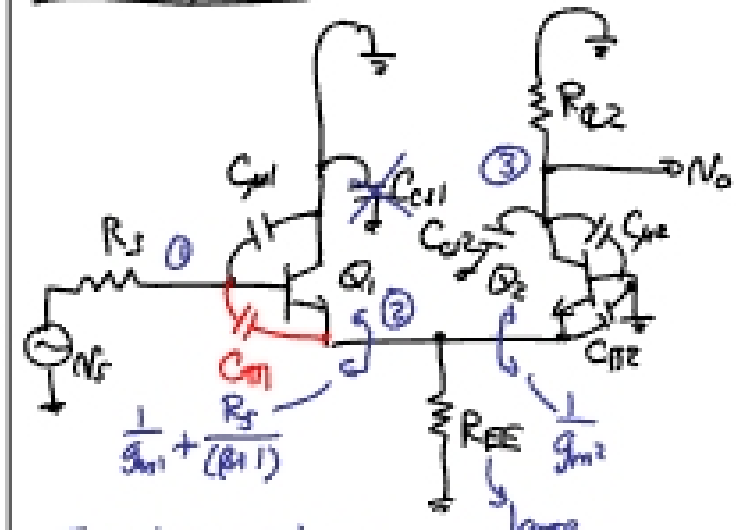
$C_\mu (1 - a_v) = C_\mu (1 + g_m R_L)$

$\tau_1 = (r_\pi \| R_s \| R_B)(C_\pi + C_\mu(1 + g_m R_L))$

$\tau_2 = R_L (C_\mu + C_{cs})$

$\omega_H = \frac{1}{\tau_1 + \tau_2}$

Multi-Stage Ex.



$\tau_1 = (2 r_\pi \| R_s) C_\mu$

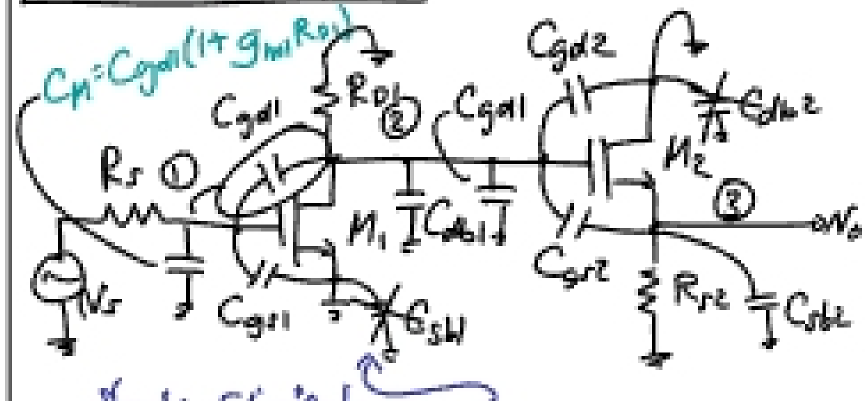
$\tau_{\pi 1} = C_{\pi 1}\left(r_{\pi 1} \| \frac{R_s + 1/g_{m2}}{1 + g_{m1}(\frac{1}{g_{m2}})}\right)$

$\tau_2 = C_{\pi 2}\left[\left(\frac{1}{g_{m1}} + \frac{R_s}{\beta + 1}\right) \| \frac{1}{g_{m2}} \| R_{EE}\right]$

$\tau_{③} = (C_{\mu 2} + C_{cs2}) R_{L2}$

$$\boxed{\omega_H = \frac{1}{\tau_{①} + \tau_{\pi 1} + \tau_{②} + \tau_{③}}}$$

MOS Two-Stage Amplifier



Step 1: Eliminate caps.

Step 2: Determine the constants.

$\tau_{①} = [C_{gs1} + C_{gd1}(1 + g_{m1}R_{D1})] R_s$

$\tau_{②} = [C_{gd1} + C_{db1} + C_{gd2}](r_o \| R_{D1})$ $\sim R_{D1}$

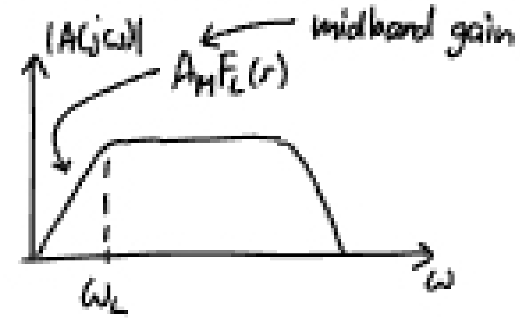
$\tau_{③} = C_{s2}\left(\frac{1}{g_{m2}+g_{mb2}} \| R_{s2}\right)$

X $\tau_{gs2} = C_{gs2}\left(\frac{R_{D1} + R_{s2}}{1 + (g_{m2}+g_{mb2})R_{s2}}\right)$

$$\boxed{\omega_H = \frac{1}{\tau_{①} + \tau_{②} + \tau_{③} + \tau_{gs2}}}$$

Low Freq. Amplifier Response Using Short Circuit Time Constant Analysis (SCTC)

Recall:



In general, for the low freq. response:

$$F_L(s) = \frac{s^{n_L} + d_1 s^{(n_L-1)} + \cdots}{s^{n_L} + e_1 s^{(n_L-1)} + \cdots}, \quad n_L = \#\text{poles} = \#\text{zeros}$$

We can express the coefficient e_1 by:

$e_1 = \omega_{P1} + \omega_{P2} + \cdots + \omega_{Pn_L}$

Similar analysis to that used for OCTC...

For the case of a dominant pole:
↳ i.e., the highest freq. pole

$$F_L(s) \cong \frac{s}{s+\omega_L} = \frac{s}{s+e_1} \rightarrow e_1 \cong \omega_{P1} = \omega_L$$

$$\therefore \boxed{\omega_L \cong e_1 = \sum_j \omega_{Pj} = \sum_j \frac{1}{C_j R_{js}} = \sum_j \frac{1}{\tau_{js}}}$$

where $C_j \triangleq$ various large (>10nF) capacitors in the ckt. (e.g., the bypass caps.)

$R_{js} \triangleq$ driving point resistance seen between the terminals of C_j determined with:

① all large capacitors short-circuited, except C_j, which is replaced by the test voltage source for R determination

For reading, can go to Sedra & Smith