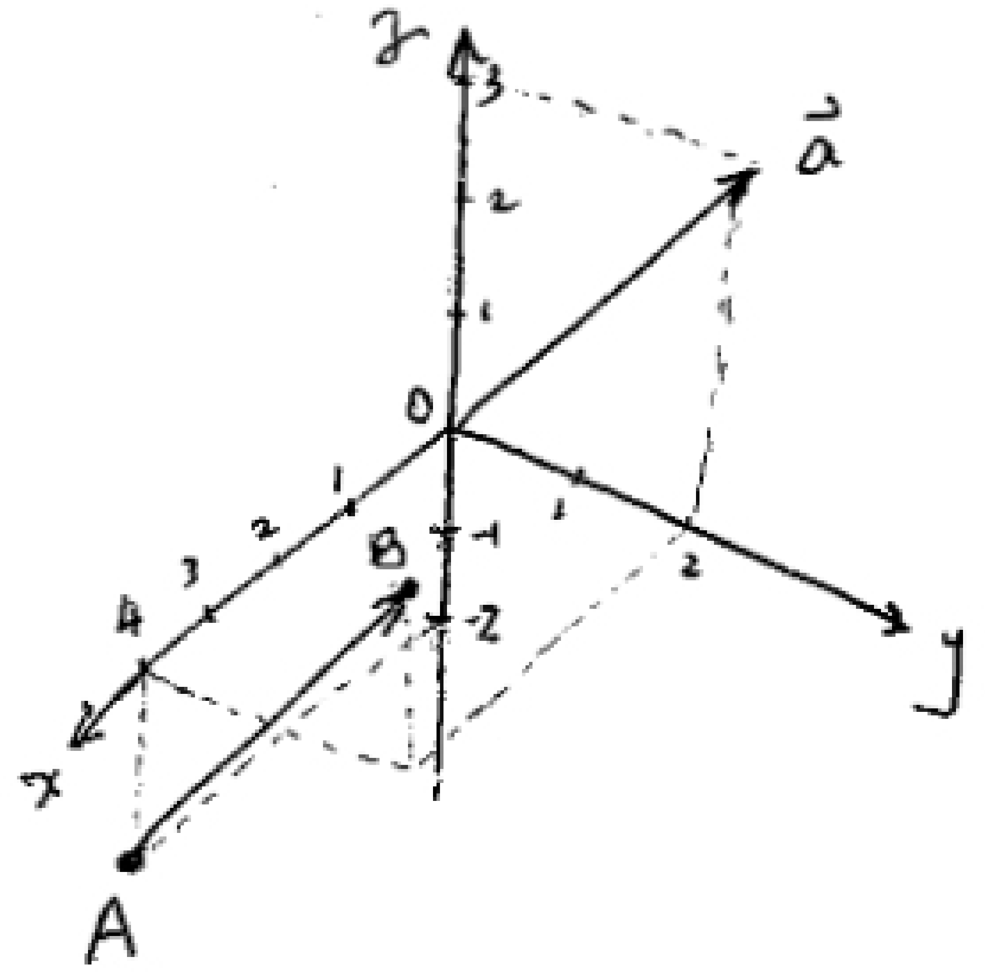
MATH 2339, HW03: Solution

§10.2

8/ (2)

$A(4,0,-2)$, $B(4,2,1)$

$$\vec{a} = \vec{AB} = \langle 4-4,\ 2-0,\ 1-(-2)\rangle = \boxed{\langle 0, 2, 3\rangle}$$ (1)



16/ (3)

$\vec{a} = 2\vec{i} - 4\vec{j} + 4\vec{k}$, $\vec{b} = 2\vec{j} - \vec{k}$

$$\vec{a}+\vec{b} = 2\vec{i} + (-4+2)\vec{j} + [4+(-1)]\vec{k}$$
$$= 2\vec{i} - 2\vec{j} + 3\vec{k}$$

$$2\vec{a}+3\vec{b} = (4\vec{i} - 8\vec{j} + 8\vec{k}) + (6\vec{j} - 3\vec{k})$$
$$= 4\vec{i} - 2\vec{j} + 5\vec{k}$$

$$|\vec{a}| = \sqrt{2^2+(-4)^2+4^2} = \sqrt{36} = 6$$

$$|\vec{a}-\vec{b}| = |2\vec{i} + (-4-2)\vec{j} + [4-(-1)]\vec{k}|$$
$$= \sqrt{2^2+(-6)^2+5^2} = \boxed{\sqrt{65}}$$ (1)

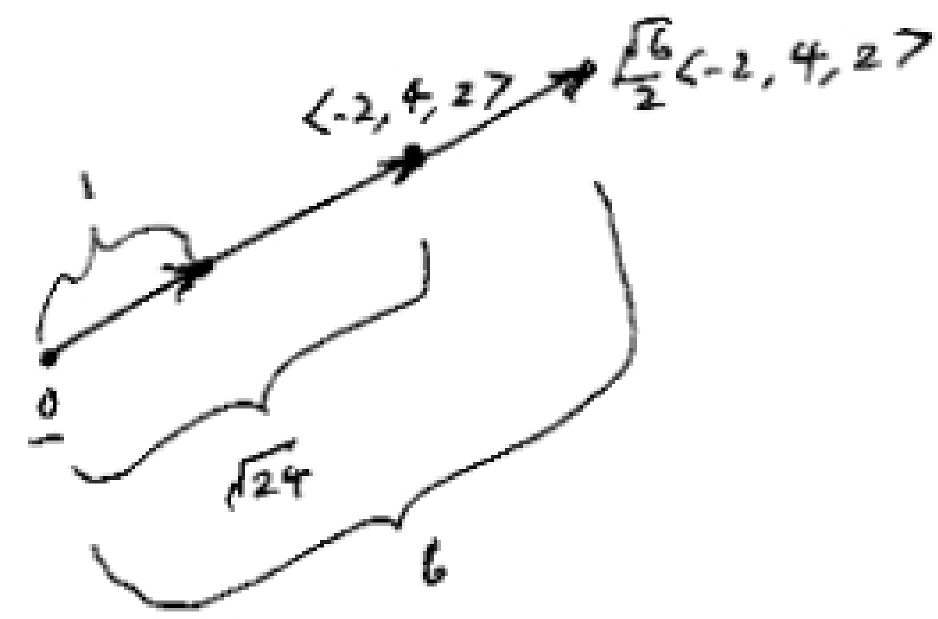
18/ ②

The unit vector in the direction of $\langle -2, 4, 2\rangle$ is :

$$\frac{\langle -2,4,2\rangle}{\sqrt{(-2)^2+4^2+2^2}} = \frac{1}{\sqrt{24}}\langle -2, 4, 2\rangle$$

The vector in the direction of $\langle -2,4,2\rangle$ with length 6 is :

$$\boxed{6\times \frac{1}{\sqrt{24}}\langle -2,4,2\rangle} = \frac{6}{2\sqrt{6}}\langle -2,4,2\rangle = \frac{\sqrt{6}}{2}\langle -2,4,2\rangle$$



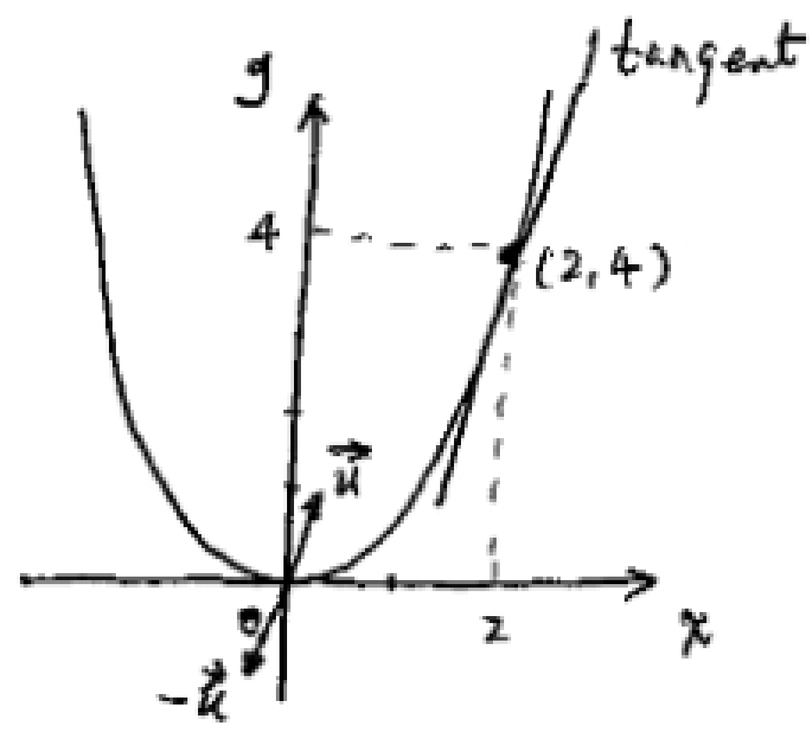
33/ ③

$y = x^2 \Rightarrow y' = 2x$

The slope of the tangent line at $(2,4)$ is :

$$y'(2) = 2\times 2 = \boxed{4}$$

So the tangent line is parallel to $\boxed{\langle 1, 4\rangle}$



The unit vectors parallel to the tangent lines are :

$$\frac{\langle 1,4\rangle}{\sqrt{1^2+4^2}} = \boxed{\frac{1}{\sqrt{17}}\langle 1,4\rangle}$$

and $\boxed{-\frac{1}{\sqrt{17}}\langle 1, 4\rangle}$.

① ②

§10.3

1/ (3)

(a) $(\vec{a}\cdot\vec{b})\cdot\vec{c}$: $\boxed{\text{meaningless}}$ $\boxed{\text{because}}$ $\vec{a}\cdot\vec{b}$ is a number and can not from the dot product between the number $\vec{a}\cdot\vec{b}$ and the vector $\vec{c}$;

(b) $(\vec{a}\cdot\vec{b})\,\vec{c}$: $\boxed{\text{meaningful}}$ $\boxed{\text{because}}$ it is a scalar multiplication of the vector $\vec{c}$ and the scalar is $\vec{a}\cdot\vec{b}$;

(c) $|\vec{a}|\,(\vec{b}\cdot\vec{c})$: $\boxed{\text{meaningful}}$ $\boxed{\text{because}}$ it is just the product of two numbers $|\vec{a}|$ and $(\vec{b}\cdot\vec{c})$;

(d) $\vec{a}\cdot(\vec{b}+\vec{c})$: $\boxed{\text{meaningful}}$ $\boxed{\text{because}}$ it is the dot product of the vector $\vec{a}$ and the vector addition $\vec{b}+\vec{c}$;

(e) $\vec{a}\cdot\vec{b}+\vec{c}$: $\boxed{\text{meaningless}}$ $\boxed{\text{because}}$ can not add a number $\vec{a}\cdot\vec{b}$ to a vector $\vec{c}$;

(f) $|\vec{a}|\cdot(\vec{b}+\vec{c})$: $\boxed{\text{meaningless}}$ $\boxed{\text{because}}$ can not form a dot product between a number $|\vec{a}|$ and a vector $\vec{b}+\vec{c}$.

(2) 8/

$$\vec{a} = 3\vec{i} + 2\vec{j} - \vec{k} = \langle 3, 2, -1\rangle$$

$$\vec{b} = 4\vec{i} + 5\vec{k} = \langle 4, 0, 5\rangle$$

$$\vec{a}\cdot\vec{b} = 3\times 4 + 2\times 0 + (-1)\times 5 = \boxed{7}$$