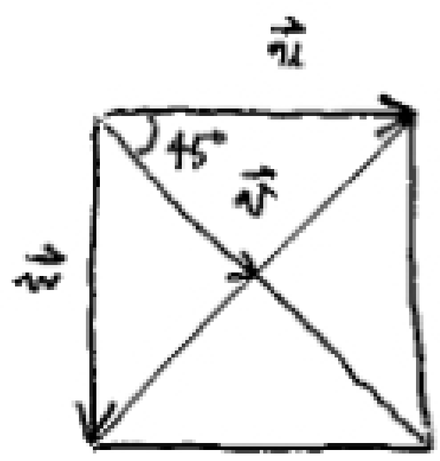
MATH 2339, HW04: Solution (13)

§10.3

2 12/.



$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos 45^\circ = 1\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{1}{2}}$$

$$\vec{u}\cdot\vec{w} = |\vec{u}||\vec{w}|\cos 90^\circ = \boxed{0}$$

2 13/.

(a) $\vec{i} = \langle 1,0,0\rangle$, $\vec{j} = \langle 0,1,0\rangle$, $\vec{k} = \langle 0,0,1\rangle$

$$\vec{i}\cdot\vec{j} = 1\times 0 + 0\times 1 + 0\times 0 = 0$$

$$\vec{j}\cdot\vec{k} = 0\times 0 + 1\times 0 + 0\times 1 = 0$$

$$\vec{k}\cdot\vec{i} = 1\times 0 + 0\times 0 + 0\times 1 = 0$$

(b) $$\vec{i}\cdot\vec{i} = 1^2+0^2+0^2 = 1$$

$$\vec{j}\cdot\vec{j} = 0^2+1^2+0^2 = 1$$

$$\vec{k}\cdot\vec{k} = 0^2+0^2+1^2 = 1$$

3 20/.

(a) $\vec{u} = \langle -3, 9, 6\rangle$, $\vec{v} = \langle 4, -12, -8\rangle$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{-3\times 4 + 9\times(-12) + 6\times(-8)}{\sqrt{(-3)^2+9^2+6^2}\sqrt{4^2+(-12)^2+(-8)^2}} = \frac{-168}{\sqrt{126}\sqrt{224}}$$

$$= -1 \Rightarrow \boxed{\theta = \pi}$$

So $\vec{u}$ and $\vec{v}$ are parallel.

(b) $\vec{u} = \vec{i} - \vec{j} + 2\vec{k}$, $\vec{v} = 2\vec{i} - \vec{j} + \vec{k}$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{1\times 2 + (-1)\times(-1) + 2\times 1}{\sqrt{1^2+(-1)^2+2^2}\sqrt{2^2+(-1)^2+1^2}} = \frac{5}{6} \Rightarrow 0 < \theta < \frac{\pi}{2}$$

So $\vec{u}$ and $\vec{v}$ are neither parallel nor orthogonal. $\sqrt{1}$

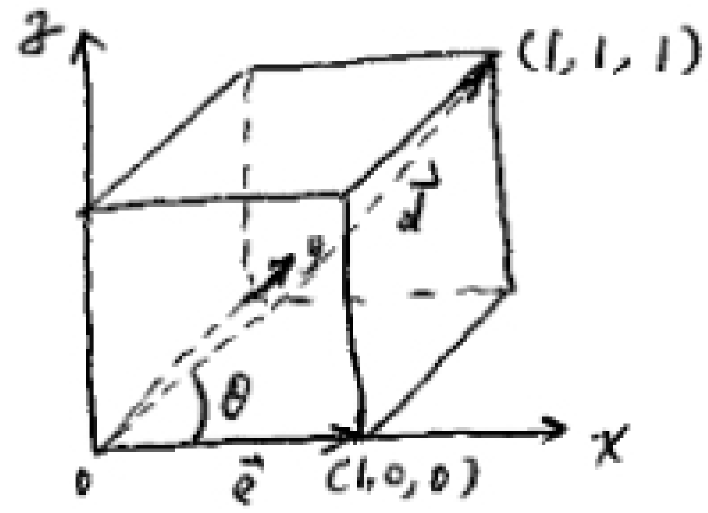
(c) $\vec{u} = \langle a, b, c \rangle$, $\vec{v} = \langle -b, a, 0 \rangle$

$\vec{u} \cdot \vec{v} = a \times (-b) + b \times a + c \cdot 0 \boxed{= 0}$

So $\vec{u}$ and $\vec{v}$ are orthogonal.

43/. ③

$\boxed{\begin{array}{l} \vec{d} = \langle 1, 1, 1 \rangle \\ \vec{e} = \langle 1, 0, 0 \rangle \end{array}}$ (diagonal) (edge)



$$\cos\theta = \frac{\vec{d} \cdot \vec{e}}{|\vec{d}||\vec{e}|} = \frac{1 \times 1 + 1 \times 0 + 1 \times 0}{\sqrt{1^2+1^2+1^2}\sqrt{1^2+0^2+0^2}}$$

$$= \frac{1}{\sqrt{3}} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{\sqrt{3}}{3}\right)} \approx 54.7^\circ$$

46/. ③

$\vec{c} = |\vec{a}|\,\vec{b} + |\vec{b}|\,\vec{a}$

$$\cos\theta_1 = \frac{\vec{c} \cdot \vec{a}}{|\vec{c}|\,|\vec{a}|} = \frac{|\vec{a}|\,\vec{b} \cdot \vec{a} + |\vec{b}|\,\vec{a} \cdot \vec{a}}{|\vec{c}|\,|\vec{a}|} = \frac{\vec{b} \cdot \vec{a} + |\vec{b}||\vec{a}|}{|\vec{c}|}$$

$$\cos\theta_2 = \frac{\vec{c} \cdot \vec{b}}{|\vec{c}||\vec{b}|} = \frac{|\vec{a}|\,\vec{b} \cdot \vec{b} + |\vec{b}|\,\vec{a} \cdot \vec{b}}{|\vec{c}|\,|\vec{b}|} = \frac{|\vec{a}||\vec{b}| + \vec{a} \cdot \vec{b}}{|\vec{c}|}$$

$\cos\theta_1 = \cos\theta_2$, $0 \le \theta_1 \le \pi$, $0 \le \theta_2 \le \pi$

So $\theta_1 = \theta_2$ and $\vec{c}$ bisects the angle between $\vec{a}$ and $\vec{b}$.