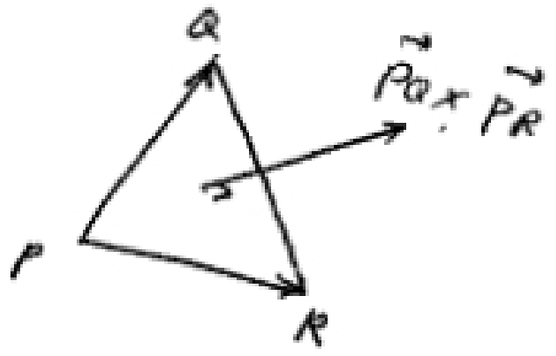
④ 30/ $P(0,0,-3)$, $Q(4,2,0)$, $R(3,3,1)$



(a) $\vec{PQ} \times \vec{PR}$ orthogonal to the plane through P, Q, R:

$$\boxed{\vec{PQ} \times \vec{PR}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 2 & 3 \\ 3 & 3 & 4 \end{vmatrix} = \langle -1, -7, 6 \rangle$$

(b) $$S_{\Delta PQR} = \boxed{\frac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|} = \frac{1}{2}\sqrt{(-1)^2 + (-7)^2 + 6^2} = \frac{1}{2}\sqrt{86}$$

② 37/ $\vec{u} = \vec{i} + 5\vec{j} - 2\vec{k}$, $\vec{v} = 3\vec{i} - \vec{j}$, $\vec{w} = 5\vec{i} + 9\vec{j} - 4\vec{k}$

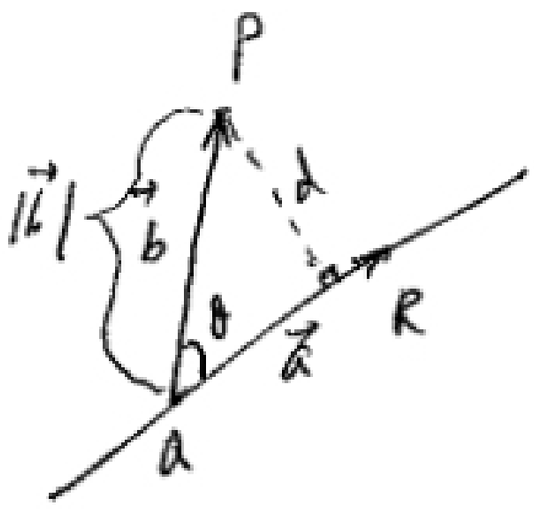
$$\boxed{\vec{u} \cdot (\vec{v} \times \vec{w})} = \langle 1, 5, -2 \rangle \cdot \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 0 \\ 5 & 9 & -4 \end{vmatrix}$$

$$= \langle 1, 5, -2 \rangle \cdot \langle 4, 12, 32 \rangle$$

$$= 4 + 60 - 64 \quad \boxed{= 0}$$

So $\vec{u}$, $\vec{v}$, $\vec{w}$ coplanar.

② 45/ (a) The distance from the point P to the line is:



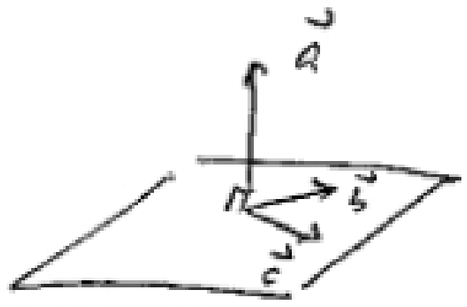
$$\boxed{\begin{aligned} d &= |\vec{b}| \sin\theta \\ &= \frac{|\vec{a}|\,|\vec{b}| \sin\theta}{|\vec{a}|} \\ &= \frac{|\vec{a} \times \vec{b}|}{|\vec{a}|} \end{aligned}}$$

53/ $\vec{a} \neq \vec{0}$

(a) $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow \vec{a} \perp (\vec{b} - \vec{c})$

but $\vec{b}$ may not equal to $\vec{c}$.

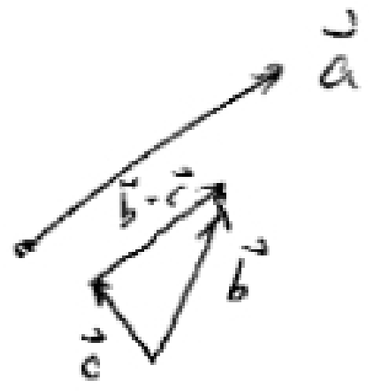


Ans: No.

(b) $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{0}$

$\Rightarrow \vec{a} \parallel (\vec{b} - \vec{c})$

Again, $\vec{b}$ may not equal to $\vec{c}$



Ans: No.

(c) $\left.\begin{array}{l} \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \\ \vec{a} \times \vec{b} = \vec{a} \times \vec{c} \end{array}\right\} \Rightarrow \vec{a} \perp (\vec{b} - \vec{c})$ and $\vec{a} \parallel (\vec{b} - \vec{c})$

$\vec{a} \neq \vec{0}$ so $\vec{b} - \vec{c} = \vec{0} \Rightarrow \vec{b} = \vec{c}$

Ans. Yes.

§ 10.5

1/

(a) True

(b) False : 

(c) True

(d) False : 

(e) False

(f) True : 

(g) False : 

(h) True

(i) True

(j) False :  (skewed)

(k) True