MATH 2339. HW08 : Solution

(14)

§10.5

18/. (3)

$L_1: \; x = 5-12t, \; y = 3+9t, \; z = 1-3t$

$L_2: \; x = 3+8s, \; y = -6s, \; z = 7+2s$

$$\boxed{\vec{v}_1 = \langle -12, 9, -3 \rangle, \quad \vec{v}_2 = \langle 8, -6, 2 \rangle}$$

$$\frac{-12}{8} = \frac{9}{-6} = \frac{-3}{2} \Rightarrow \boxed{\vec{v}_1 = -\frac{3}{2}\vec{v}_2}$$

So L_1 and L_2 **parallel.**

Are L_1 and L_2 the same line?

$$\left.\begin{cases} 5-12t = 3+8s \\ 3+9t = -6s \\ 1-3t = 7+2s \end{cases}\right\} \Rightarrow 6 = 21 \text{, which is impossible}$$

So L_1 and L_2 have no common points and are different lines which are parallel.

23/. (2)

$P_0(1,-1,-1)$, $\boxed{\vec{n} = \langle 5, -1, -1 \rangle}$ ← coefficients of x, y, z in the eqn. of the parallel plane

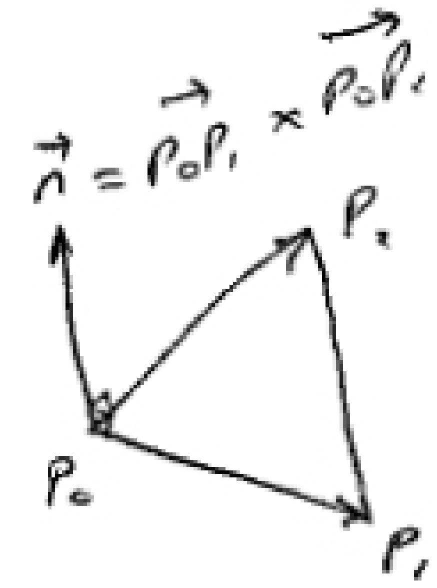
So: $$\boxed{5(x-1) + (-1)(y-(-1)) + (-1)(z-(-1)) = 0}$$

i.e. $$5(x-1) - (y+1) - (z+1) = 0$$

or $$5x - y - z - 7 = 0$$

26/. (3)

$P_0(0,0,0)$, $P_1(2,-4,6)$, $P_2(5,1,3)$



$$\boxed{\vec{n} = \vec{P_0P_1} \times \vec{P_0P_2}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -4 & 6 \\ 5 & 1 & 3 \end{vmatrix}$$

$$= \boxed{-18\vec{i} + 24\vec{j} + 22\vec{k}}$$

So: $\boxed{-18(x-0) + 24(y-0) + 22(z-0) = 0}$

i.e. $-18x + 24y + 22z = 0$

36/. (2) $\vec{n}_1 = \langle 2, -3, 4\rangle$, $\vec{n}_2 = \langle 1, 6, 4\rangle$

Since $\frac{2}{1} \neq \frac{-3}{6}$, $\vec{n}_1$ is not parallel to $\vec{n}_2$

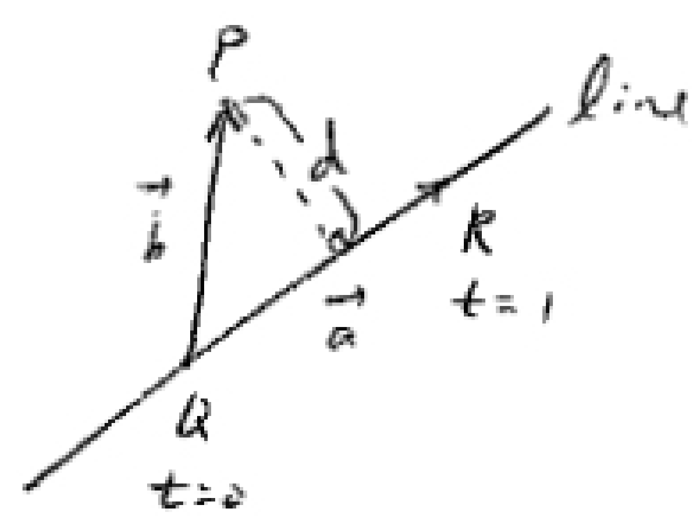
and the two planes are not parallel. They intersect.

The angle between the planes:

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{2-18+16}{|\vec{n}_1||\vec{n}_2|} = 0 \Rightarrow \boxed{\theta = 90^\circ}$$

So the two planes are $\boxed{\text{perpendicular}}$

48/. (4) $\underbrace{(0,1,3)}_{P}$; $x = 2t,\ y = 6-2t,\ z = 3+t$



Find two points on the line.

Let $t=0$: $Q(0,6,3)$

$t=1$: $R(2,4,4)$

So: $\vec{a} = \vec{QR} = \langle 2,-2,1\rangle$

$\vec{b} = \vec{QP} = \langle 0,-5,0\rangle$

The distance from P to the line:

$$d = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|}, \text{ where } |\vec{a}| = \sqrt{2^2+(-2)^2+1} = \sqrt{9} = 3$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -2 & 1 \\ 0 & -5 & 0 \end{vmatrix} = 5\vec{i} - 10\vec{k}, \quad |\vec{a}\times\vec{b}| = \sqrt{5^2+10^2} = 5\sqrt{5}$$

So $\boxed{d = 5\sqrt{5}/3}$.

20/. $L_1: \frac{x}{1} = \frac{y-1}{-1} = \frac{z-2}{3}$

$L_2: \frac{x-2}{2} = \frac{y-3}{-2} = \frac{z}{7}$

The directions:

$L_1: \quad \vec{v}_1 = \langle 1, -1, 3 \rangle$

$L_2: \quad \vec{v}_2 = \langle 2, -2, 7 \rangle$

$\vec{v}_1$ and $\vec{v}_2$ are not parallel (one is not the scalar multiple of the other), so L_1 and L_2 are not parallel.

Assume L_1 and L_2 intersect. Then:

$$L_1: \begin{cases} x = 0 + t \\ y = 1 + (-1)t \\ z = 2 + 3t \end{cases} \qquad L_2: \begin{cases} x = 2 + 2s \\ y = 3 + (-2)s \\ z = 0 + 7s \end{cases}$$

So: $\begin{cases} t = 2 + 2s & ① \\ 1 - t = 3 - 2s & ② \\ 2 + 3t = 7s & ③ \end{cases}$ ①② $\Rightarrow 1 - (2 + 2s) = 3 - 2s$

i.e. $-1 - 2s = 3 - 2s$

i.e. $-1 = 3$ impossible

The assumption is wrong. L_1 and L_2 are skew lines.