12

MATH 2339, HW09: Solution

§10.7

2 2/. $\vec{r}(t) = \frac{t-2}{t+2}\vec{i} + \sin t\,\vec{j} + \ln(9-t^2)\,\vec{k}$

domain of $f(t) = \frac{t-2}{t+2}$: $t \neq -2$

domain of $g(t) = \sin t$: $-\infty < t < \infty$ (i.e. $t \in \mathbb{R}$)

domain of $h(t) = \ln(9-t^2)$: $9 - t^2 > 0 \Rightarrow 9 > t^2 \Rightarrow$

$3 > t > -3$

So domain of $\vec{r}(t)$: $\boxed{-3 < t < 3 \quad \text{and} \quad t \neq -2}$

3 4/. $\lim_{t\to 1} \frac{t^2-t}{t-1} = \lim_{t\to 1}\frac{t(t-1)}{t-1} = \lim_{t\to 1} t = \boxed{1}$

$\lim_{t\to 1}\sqrt{t+8} = \sqrt{1+8} = \boxed{3}$

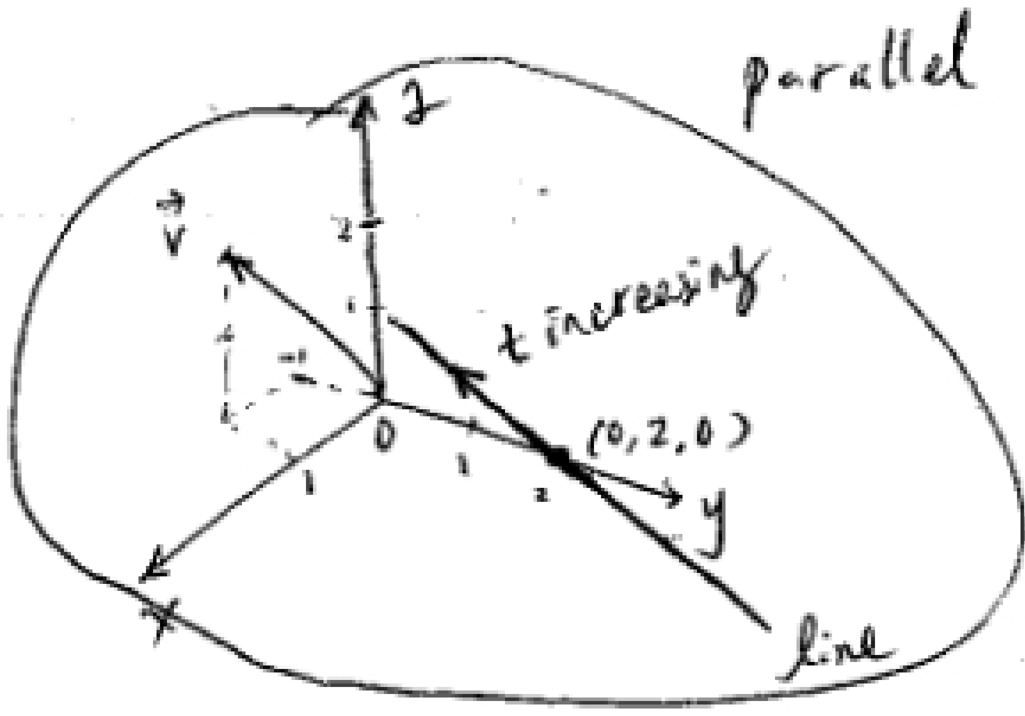
$\lim_{t\to 1}\frac{\sin \pi t}{\ln t} \underset{\text{L'H}}{\overset{0/0}{=}} \lim_{t\to 1}\frac{\pi\cos\pi t}{(1/t)} = \frac{\pi\cos\pi}{1} = \boxed{-\pi}$

So: $\lim_{t\to 1}\left(\frac{t^2-t}{t-1}\vec{i} + \sqrt{t+8}\,\vec{j} + \frac{\sin\pi t}{\ln t}\vec{k}\right) = \boxed{(\vec{i} + 3\vec{j} - \pi\vec{k})}$

3 7/. $\vec{r}(t) = \langle t, 2-t, 2t\rangle$

$x = t, \quad y = 2-t, \quad z = 2t$: line through $\boxed{(0,2,0)}$

parallel to $\boxed{\vec{v} = \langle 1, -1, 2\rangle}$

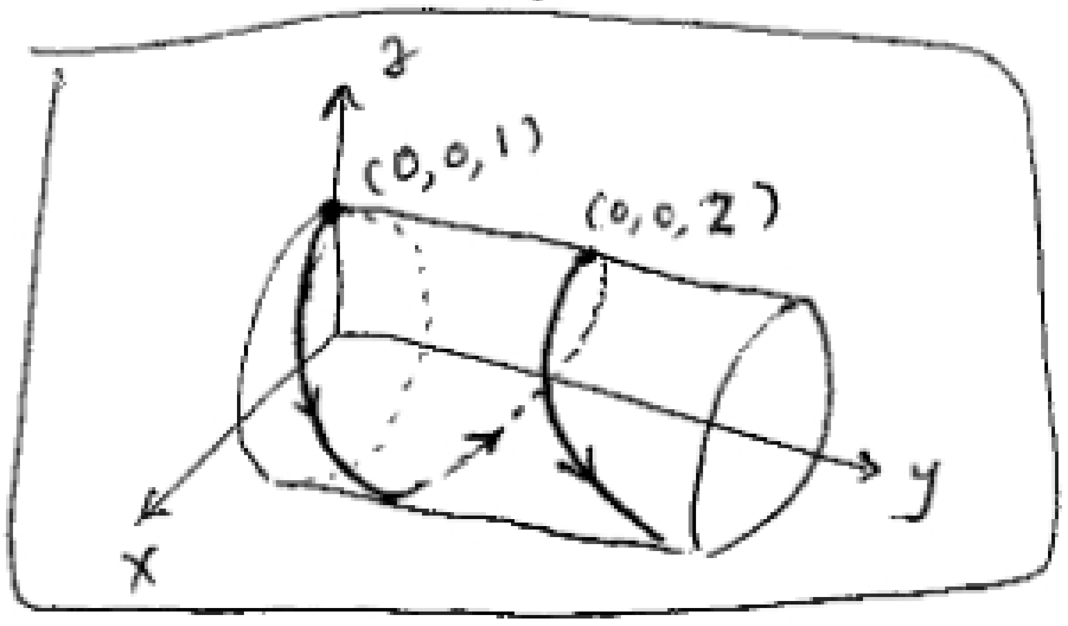


8/. (2)

$\vec{r}(t) = \langle \sin\pi t,\ t,\ \cos\pi t \rangle$

$x = \sin\pi t,\ y = t,\ z = \cos\pi t$

$x^2 + z^2 = 1 \Rightarrow$ curve on the cylinder $x^2+z^2=1$
y increases linearly w/. t
$\Big\} \Rightarrow \boxed{\text{a helix}}$



17/. (2)

$x = t\cos t,\ y = t,\ z = t\sin t, \quad t \geq 0$

$x^2 + z^2 = t^2 \Rightarrow$ the curve is on a $\boxed{\text{cone}}$

(radius increases from $t^2=0^2$ to t^2)

y increases linearly w/. t.

So the curve spirals on the cone toward the positive y direction. $\boxed{\text{The graph is II.}}$