Homework assignment 1 in MATH 309

Solutions

1. (a) $2x_1+x_2=6$ is a line passing through

(3,0) and (0,6) (this are intercepts with the axis

$x_1-3x_2=-4$ is a line passing through

$(-4,0)$ and $(0,\frac{4}{3})$

This two lines are not parallel

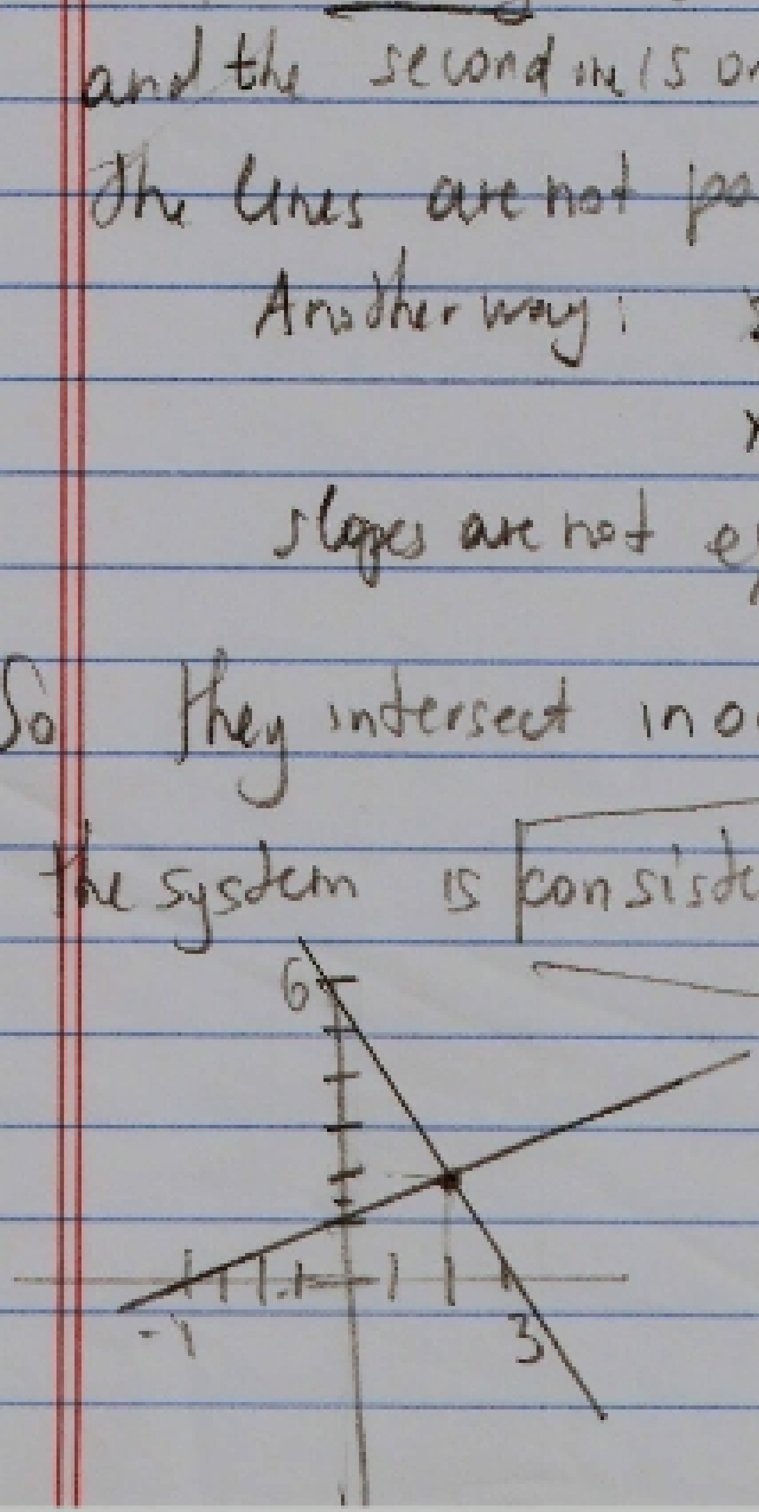
Explanation: <u>One way</u>: The first line is orthogonal to the vector $(2,1)$ and the second one is ortogonal to $(1,-3)$; $(2,1) \nparallel (1,-3) \Rightarrow$ the lines are not parallel

Another way: $2x_1+x_2=6 \Rightarrow x_2=-2x_1+6 \Rightarrow$ slope $=-2$

$x_1-3x_2=-4 \Rightarrow x_2=\frac{1}{3}x_1+\frac{4}{3} \Rightarrow$ slope $=\frac{1}{3}$

slopes are not equal $\Rightarrow$ the lines are not parallel

So they intersect in one point $\Rightarrow$ there is $\boxed{\text{one solution}}$ $\Rightarrow$ the system is $\boxed{\text{consistent}}$



Algebraic solution

$2x_1+x_2=6$

$x_1-3x_2=-4$

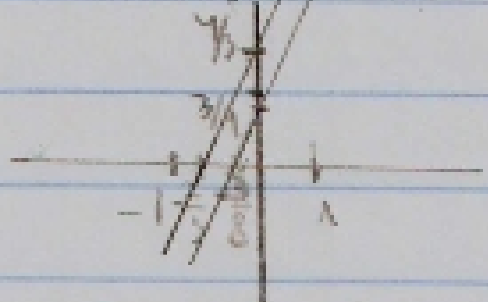
Eq 1 − 2 Eq 2 : $7x_2=14 \Rightarrow x_2=2 \Rightarrow$

$2x_1+2=6 \Rightarrow x_1=2 \Rightarrow$

Solution is $\boxed{(2,2)}$

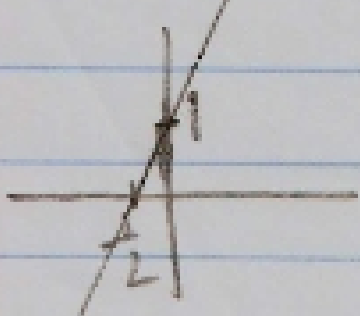
(b) $\begin{cases} 6x_1 - 3x_2 = -4 \iff 2x_1 - x_2 = -\frac{4}{3} \iff x_2 = 2x_1 + \frac{4}{3} \\ -8x_1 + 4x_2 = 3 \iff 2x_1 - x_2 = -\frac{3}{4} \iff x_2 = 2x_1 + \frac{3}{4} \end{cases}$

Two parallel lines (orthogonal to the vector $(2,-1)$) or with slope 2) which do not coincide) $\Rightarrow$ the system is $\boxed{\text{Inconsistent}}$



(c) $\begin{cases} 6x_1 - 3x_2 = -3 \\ -8x_1 + 4x_2 = 4 \end{cases} \iff \begin{cases} 2x_1 - x_2 = -1 \\ 2x_1 - x_2 = -1 \end{cases} \Rightarrow x_2 = 2x_1 + 1$

$\rightarrow$ Two lines coinside $\Rightarrow$ $\boxed{\text{Consistent}}$ $\boxed{\text{Infinite many solutions}}$ $\boxed{(\alpha, 2\alpha+1)}$



Problem 2 From the last equation: $7x_5 = -7 \Rightarrow \boxed{x_5 = -1}$

Substitute x_5 to the Eq 4: $10x_4 - 5 = 5 \Rightarrow 10x_4 = 10 \Rightarrow \boxed{x_4 = 1}$

Substitute x_4 & x_5 into Eq 3: $3x_3 + 5 - 1 \Rightarrow x_3 = -\frac{4}{3}$

Substitute x_3, x_4 & x_5 into Eq 2: $2x_2 + \frac{4}{3} - 2 + 4 = -4 \Rightarrow$

$$2x_2 = -6 - \frac{4}{3} = -\frac{22}{3} \Rightarrow x_2 = -\frac{11}{3}$$

Substitute: x_2, x_3, x_4 & x_5 into Eq 1

$$3x_1 - \frac{55}{3} - \frac{8}{3} - 4 + 1 = 4$$

$$3x_1 = \frac{63}{3} + 7 = 21 + 7 = 28 \Rightarrow x_1 = \frac{28}{3}$$

Answer: $\left(\frac{28}{3}, -\frac{11}{3}, -\frac{4}{3}, 1, -1\right)$

Problem 3 (Section 1.2, p.23, problem 1)

(a) It is in row echelon form but not in the reduced row echelon form, because above the leading coefficient of the second row there is a nonzero entry in the same column

(b) It is not in row echelon form (and therefore not in reduced row echelon form), because row three is nonzero row and its below the row two, which consists of zeros

(c) It is in row echelon and in the reduced row echelon forms

(d) It is in row echelon and in the reduced row echelon forms

(e) It is not in row echelon form (and therefore not in the reduced row echelon form), because the first nonzero entry of the third row is 3

(f) It is not in row echelon form (and therefore not in reduced row echelon form) because # of leading zeros in the 3rd row is 1 and less than # of leading zeros in the second column

(g) It is in row echelon and in the reduced row echelon form