Problem 1

Table 1 shows the given data.

TABLE 1—GIVEN DATA	
Parameter	Value
Horizontal permeability, md	8.2
Reservoir thickness, ft	53
Initial reservoir pressure, psi	5,651
Bubble point pressure, psi	1,323
Total compressibility, psi^{-1}	1.29 x 10^{-5}
Oil viscosity, cp	1.7
Formation volume factor, RB/STB	1.1
Porosity	0.19
Wellbore radius, ft	0.328
Drainage area, acres	40
Bottomhole flowing pressure, psi	3,000

Assuming that the shape of the drainage area is circular and the well is located in the center, we have

$$A = \pi r_e^2 \quad \text{......(1.1)}$$

where r_e is the external drainage radius in ft. Solving for r_e, we obtain

$$r_e = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{(40\,\text{acres})\left(43{,}560\,\dfrac{\text{ft}}{\text{acre}}\right)}{\pi}} \cong 745\text{ ft.} \quad \text{......(1.2)}$$

We assume that there is no transition period between transient flow and pseudo-steady-state flow periods. From Economides *et al*., Petroleum Production System (p. 25, Eq. 2-35), the time, at which pseudo-steady state begins, is given by

$$t_{pss} = 1200\frac{\phi\mu c_t r_e^2}{k} = \frac{(1200)(0.19)(1.7)(1.29\times10^{-5})(745)^2}{8.2} \cong 338\,\text{hrs.} \quad \text{......(1.3)}$$

Hence, pseudo-steady-state will be reached before two-year production.

<u>Case 1: $s = 0$</u>

(i) $t \le t_{pss}$: Transient flow period

From Economides *et al*., Petroleum Production System (p. 18, Eq. 2-7), transient flow equation is given by

$$q = \frac{kh(p_i - p_{wf})}{162.6B\mu}\left(\log\frac{k}{\phi\mu c_t r_w^2}t - 3.23 + 0.869s\right)^{-1} \quad \text{......(1.4)}$$

However, we cannot integrate Eq. 1.4 with respect to t analytically. But, with the trapezoidal rule, we can calculate the cumulative production at t, $Q(t)$, approximately as follows (**Fig. 1.1**):

$$Q(t) = \sum_i\left[\frac{q(t_{i+1}) + q(t_i)}{2}(t_{i+1} - t_i)\right] \quad \text{......(1.5)}$$

$$Q(t = t_{pss}) \cong 8{,}801\,\text{STB.}$$

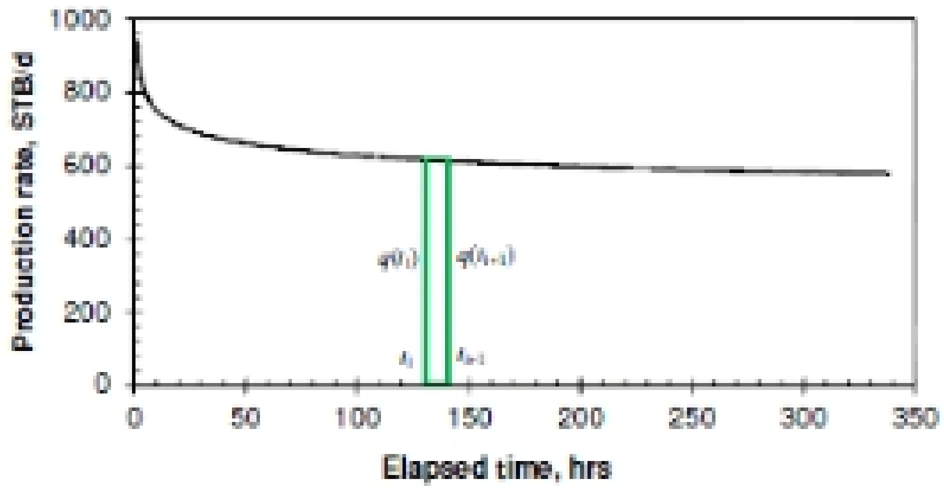

Fig. 1.1—Production rate vs. elapsed time.

(ii) $t > t_{pss}$: Pseudo-steady-state flow period
From Economides *et al.*, Petroleum Production System (p. 23, Eq. 2-7), pseudo-steady-state flow equation is given by

$$q = \frac{kh(\overline{p}(t) - p_{wf})}{141.2B\mu}\left(\ln\frac{0.472r_e}{r_w} + s\right)^{-1} \quad \text{.......(1.6)}$$

First, we calculate the average reservoir pressure at $t = t_{pss}$. Combining Eqs. 1.4 and 1.6 and solving for the average pressure gives

$$\overline{p}(t_{pss}) = p_{wf} + \frac{(141.2)(p_i - p_{wf})\left(\ln\frac{0.472r_e}{r_w} + s\right)}{(162.6)\left(\log\frac{k}{\phi\mu c_t r_w^2}t_{pss} - 3.23 + 0.869s\right)} \quad \text{.......(1.7)}$$

$$\overline{p}(t_{pss}) = 3{,}000 + \frac{(141.2)(5{,}651 - 3{,}000)\left[\ln\frac{(0.472)(745)}{(0.328)} + 0\right]}{(162.6)\left[\log\frac{(8.2)(338)}{(0.19)(1.7)(1.29\times10^{-5})(0.328)^2} - 3.23 + (0.869)(0)\right]}$$

$\cong$ 5,448 psi.

From the condition of the problem, average reservoir pressure is given by

$$\overline{p}(t) = \overline{p}(t_{pss}) - \frac{500}{(365)(24)}(t - t_{pss}) \cong \overline{p}(t_{pss}) - 0.057(t - t_{pss}) \quad \text{.......(1.8)}$$

where t is in hours. At t = 17,520 hrs (= 2 years),

$\overline{p}(17{,}520) = 5{,}448 - (0.057)(17{,}520 - 338) \cong 4{,}469$ psi.

Even after two years, the average reservoir pressure is higher than bubble point pressure (1,323 psi). Hence, Eq. 1.6 is valid for this problem.

The cumulative production at t, $Q(t)$, is given by

$$Q(t) = Q(t_{pss}) + \int_{t_{pss}}^{t}\frac{1}{24}\cdot\frac{kh[\overline{p}(t) - p_{wf}]}{141.2B\mu}\left(\ln\frac{0.472r_e}{r_w} + s\right)^{-1}dt\,. \quad \text{.......(1.9)}$$

Substituting Eq. 1.8 into Eq. 1.9 gives

$$Q(t) = Q(t_{pss}) + \frac{1}{24}\cdot\frac{kh}{141.2B\mu}\left(\ln\frac{0.472r_e}{r_w}+s\right)^{-1} \times\left\{\left[\overline{p}(t_{pss})+0.057t_{pss}-p_{wf}\right](t-t_{pss})-\frac{0.057}{2}(t^2-t_{pss}^2)\right\} \quad \text{.......(1.10)}$$

For example,

$$Q(17{,}520) = 8{,}801+\frac{1}{24}\cdot\frac{(8.2)(53)}{(141.2)(1.1)(1.7)}\left(\ln\frac{(0.472)(745)}{0.328}+0\right)^{-1} \times\left\{\left[5{,}448+(0.057)(338)-3{,}000\right](17{,}520-338)-\frac{0.057}{2}(17{,}520^2-338^2)\right\}$$

$$= 339{,}523 \text{ STB}.$$

The production rate and the cumulative production as functions of time are plotted in **Fig. 1.2**.

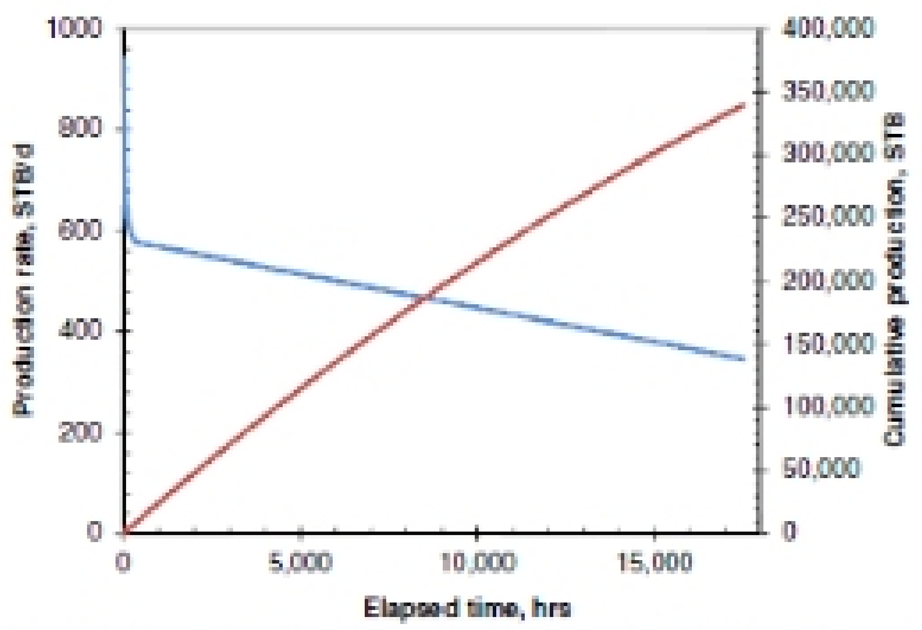

Fig. 1.2—Production rate and cumulative production vs. time with $s = 0$.

Now we solve the time when the half of the two year's cumulative production has been produced:

$$\frac{1}{2}Q(17{,}520) = Q(t_{pss}) + \frac{1}{24}\cdot\frac{kh}{141.2B\mu}\left(\ln\frac{0.472r_e}{r_w}+s\right)^{-1} \times\left\{\left[\overline{p}(t_{pss})+0.057t_{pss}-p_{wf}\right](t-t_{pss})-\frac{0.057}{2}(t^2-t_{pss}^2)\right\}$$

or

$$-0.00028t^2+24.251245t-169{,}130.2=0.$$

Solving for t, we obtain **t = 7,650 hrs (318.8 days)**.