$D/4 = 1+3 = 4$ or $3a^2-2a-1 = (3a+1)(a-1) = 0$

$a_1 = \frac{1+2}{3} = 1$

$a_2 = \frac{1-2}{3} = -\frac{1}{3}$

Answer $a = \boxed{1 \text{ or } -\frac{1}{3}}$

Problem 6

(a) $u_{tt} - tu_x = 0$

$A = 1, B = 0, C = 0 \Rightarrow B^2 - 4AC = 0 \Rightarrow$ the equation is $\boxed{\text{parabolic}}$

(b) $u_{tt} + 4u_{xt} + 4u_{xx} + x^2 u_x - t^2 u_y + \sin(tx)u = 0$

$A = 1, B = 4, C = 4 \Rightarrow$

$B^2 - 4AC = 16 - 16 = 0 \Rightarrow$ the equation is $\boxed{\text{parabolic}}$

(c) $u_{xx} + xu_{xy} + yu_{yy} = 0$

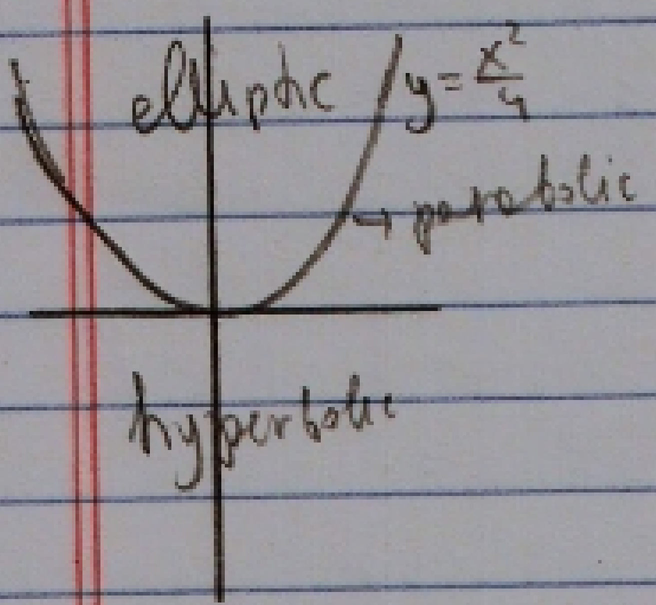
$A = 1, B = x, C = y \Rightarrow$

$B^2 - 4AC = x^2 - 4y \Rightarrow$

$\Rightarrow$ • If $x^2 - 4y > 0 \Leftrightarrow y < \frac{x^2}{4}$ then at these points the equation is <u>hyperbolic</u>

• If $x^2 = 4y \Leftrightarrow y = \frac{x^2}{4}$, then on this parabola the equation is <u>parabolic</u>

If $x^2 - 4y < 0 \Leftrightarrow y > \frac{x^2}{4}$, then at these points the equation is elliptic



<u>Problem 7</u> (a) $u_{xy} = u$

Look for the solution in the form $u = X(x)Y(y)$

$$u_{xy} = X'(x)Y'(y) \Rightarrow$$

$$X'Y' = XY \Rightarrow \underbrace{\frac{X'}{X}}_{\text{depends on } x} = \underbrace{\frac{Y}{Y'}}_{\text{depends}} = c \Rightarrow \boxed{\begin{cases} X' = cX \\ Y' = \frac{1}{c} Y \end{cases}}$$

The variables in the equation separate.

$u_{tt} + 2u_t - 4u_{xx} + u = 0$

(b) Let $u = T(t)X(x) \Rightarrow$ substituting into equation

we get

$$T''X + 2T'X - 4TX'' + TX = 0$$

Dividing by XT.

$$\frac{T''}{T} + \frac{2T'}{T} - 4\frac{X''}{X} + 1 = 0 \Rightarrow$$

$$\underbrace{\frac{T''}{T} + \frac{2T'}{T} + 1}_{\text{depends on } t \text{ only}} = \underbrace{\frac{4X''}{X}}_{\text{depends on } x \text{ only}} = c \Rightarrow$$

$$\boxed{\begin{cases} 4X'' = cX \\ T'' + 2T' + (1-c)T = 0 \end{cases}}$$

The variables in the equation separate

(c) $(x^2+t^2)u_{tt} = u_{xx}$

Let $u = T(t)X(x) \Rightarrow$

$$\underbrace{(x^2+t^2)}_{\text{cannot be seperated because of this term}} T''X = TX''$$