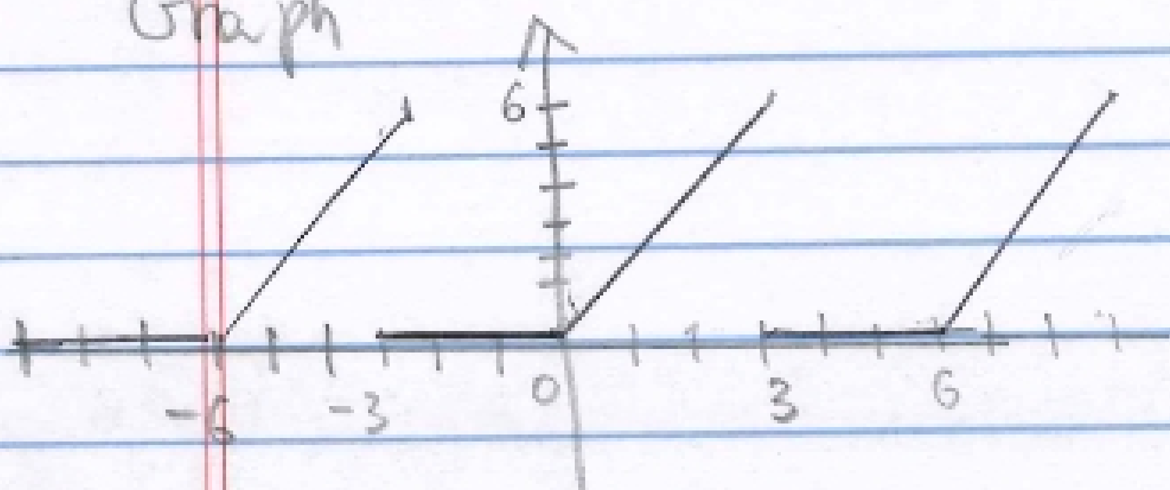
Homework #11 Solutions MATH309 Spring 2013

Problem 1

a) Graph



Find the coefficients of the Fourier series

$L = 3$

For $n \geq 1$ $a_n = \frac{1}{3}\int_{-3}^{3} f(x)\cos\frac{\pi n x}{3}\,dx = \frac{1}{3}\int_{0}^{3} 2x\cos\frac{\pi n x}{3}\,dx \underset{\text{by parts}}{\overset{\text{integration}}{=}}$

$$= \underbrace{\frac{2}{3}x\frac{\sin\frac{\pi n x}{3}}{\frac{\pi n}{3}}\Big|_0^3}_{0} - \frac{2}{3\frac{\pi n}{3}}\int_0^3 \sin\frac{\pi n x}{3}\,dx =$$

$$= \frac{2}{\frac{\pi^2 n^2}{3}}\cos\frac{\pi n x}{3}\Big|_0^3 = \boxed{\frac{6}{\pi^2 n^2}(\cos\pi n - 1)} = \frac{6}{\pi^2 n^2}((-1)^n - 1)$$

$$= \begin{cases} 0 & n \text{ is odd} \\ -\frac{12}{\pi^2 n^2} & n \text{ is even} \end{cases}$$

$$a_0 = \frac{1}{3}\int_{-3}^{3} f(x)\,dx = \frac{1}{3}\int_0^3 2x\,dx = \frac{2}{3}\cdot\frac{x^2}{2}\Big|_0^3 = 3$$

$$b_n = \frac{1}{3}\int_{-3}^{3} f(x)\sin\frac{\pi n x}{3}\,dx = \frac{1}{3}\int_{0}^{3} 2x \sin\frac{\pi n x}{3} =$$

$$= -\frac{2}{3}\,\frac{x\cos\frac{\pi n x}{3}}{\frac{\pi n}{3}}\Bigg|_0^3 + \frac{2}{3\,\frac{\pi n}{3}}\underbrace{\int_0^3 \cos\frac{\pi n x}{3}}_{0} =$$

$$= -\frac{2}{\pi n}\,3\cos\pi n = \boxed{-\frac{6}{\pi n}\cos\pi n} = -6\,\frac{(-1)^n}{\pi n}$$

The Fourier series is

$$\boxed{\frac{3}{2} + \sum_{n=1}^{\infty}\left(\frac{6}{\pi^2 n^2}(\cos\pi n - 1)\cos\frac{\pi n x}{3} - \frac{6}{\pi n}\cos\pi n \sin\frac{\pi n x}{3}\right)} =$$

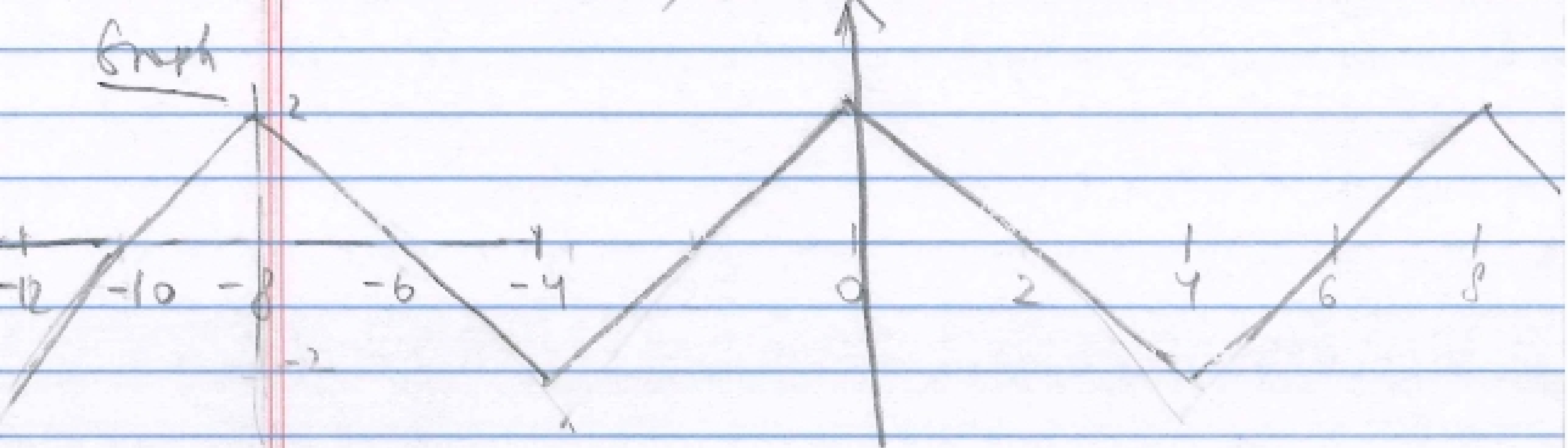
$$= \frac{3}{2} - \frac{12}{\pi^2}\sum_{k=1}^{\infty}\frac{\cos\frac{\pi(2k-1)x}{3}}{(2k-1)^2} - \frac{6}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{\pi n x}{3} =$$

Substitution: $n = 2k-1$

All possible, equivalent answers

(b) $f(x) = \begin{cases} 2-x, & 0 \le x \le 4 \\ x+2, & -4 < x < 0 \end{cases}$ period 8

Graph



This function is even $\Rightarrow b_n = 0$

$\underline{L=4}$

$n \geq 1 \quad a_n = \frac{1}{4}\int_{-4}^{4} f(x)\cos\frac{\pi n x}{4}\,dx = \frac{2}{4}\int_0^4 f(x)\cos\frac{\pi n x}{4}\,dx =$

$= \frac{1}{2}\int_0^4 (2-x)\cos\frac{\pi n x}{4}\,dx = \underbrace{\frac{(2-x)}{2}\frac{\sin\frac{\pi n x}{4}}{\frac{\pi n}{4}}\Big|_0^4}_{0} +$

$+ \frac{2}{\pi n}\int_0^4 \sin\frac{\pi n x}{4}\,dx = -\frac{2}{\frac{\pi^2 n^2}{4}}\cos\frac{\pi n x}{4}\Big|_0^4 = \frac{8}{\pi^2 n^2}(1-\cos \pi n)$

$\frac{8}{\pi^2 n^2}(1-(-1)^n) = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{16}{\pi^2 n^2} & \text{if } n \text{ is odd} \end{cases}$

$a_0 = \frac{2}{4}\int_0^4 (2-x)\,dx = \frac{1}{2}\left(2x - \frac{x^2}{2}\right)\Big|_0^4 = \frac{1}{2}\left(8 - \frac{16}{2}\right) = 0$