MATH 2339, HW14: Solution (14)

§11.3

8/. (2)

$f(x,y) = x^4y^3 + 8x^2y$

$f_x(x,y) = \boxed{4x^3y^3 + 16xy}$ (1)

$f_y(x,y) = \boxed{x^4\, 3y^2 + 8x^2 = 3x^4y^2 + 8x^2}$ (1)

9/. (2)

$f(x,t) = e^{-t}\cos\pi x$

$f_x(x,t) = e^{-t}(-\sin\pi x)\cdot\pi = \boxed{-\pi e^{-t}\sin\pi x}$ (1)

$f_t(x,t) = \boxed{-e^{-t}\cos\pi x}$ (1)

10/. (2)

$f(x,t) = \sqrt{x}\,\ln t$

$f_x(x,t) = \frac{1}{2}x^{-\frac{1}{2}}\ln t = \boxed{\dfrac{\ln t}{2\sqrt{x}}}$ (1)

$f_t(x,t) = \sqrt{x}\cdot\frac{1}{t} = \boxed{\dfrac{\sqrt{x}}{t}}$ (1)

49/. (4)

$x^2 - y^2 + z^2 - 2z = 4$

$\frac{\partial}{\partial x}\left(x^2 - y^2 + [z(x,y)]^2 - 2[z(x,y)]\right) = 0$

steps (1)

$\Rightarrow\ 2x - 0 + 2z\frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial x} = 0$

$\Rightarrow\ (2z-2)\frac{\partial z}{\partial x} = -2x \ \Rightarrow\ \frac{\partial z}{\partial x} = \frac{-2x}{2z-2} = \boxed{\dfrac{x}{1-z}}$ (1)

$\frac{\partial}{\partial y}\left(x^2 - y^2 + [z(x,y)]^2 - 2[z(x,y)]\right) = 0$

steps (1)

$\Rightarrow\ 0 - 2y + 2z\frac{\partial z}{\partial y} - 2\frac{\partial z}{\partial y} = 0$

$\Rightarrow\ (2z-2)\frac{\partial z}{\partial y} = 2y \ \Rightarrow\ \frac{\partial z}{\partial y} = \frac{2y}{2z-2} = \boxed{\dfrac{y}{z-1}}$ (1)

41/ $e^z = xyz$

(4)

$$\frac{\partial}{\partial x}\left[e^{z(x,y)}\right] = \frac{\partial}{\partial x}\left[xy\,z(x,y)\right]$$

$$\Rightarrow e^z \cdot \frac{\partial z}{\partial x} = y\frac{\partial}{\partial x}\left[x \cdot z(x,y)\right] = y\left[x\frac{\partial z}{\partial x} + z\right] = xy\frac{\partial z}{\partial x} + yz$$

$$\Rightarrow (e^z - xy)\frac{\partial z}{\partial x} = yz$$

✓ steps (1)

$$\Rightarrow \frac{\partial z}{\partial x} = \boxed{\frac{yz}{e^z - xy}}$$

$$\frac{\partial}{\partial y}\left[e^{z(x,y)}\right) = \frac{\partial}{\partial y}\left[xy\,z(x,y)\right]$$

$$\Rightarrow e^z \cdot \frac{\partial z}{\partial y} = x\frac{\partial}{\partial y}\left[y\,z(x,y)\right] = x\left[y\frac{\partial z}{\partial y} + z\right] = xy\frac{\partial z}{\partial y} + xz$$

$$\Rightarrow (e^z - xy)\frac{\partial z}{\partial y} = xz$$

✓ steps (1)

$$\Rightarrow \frac{\partial z}{\partial y} = \boxed{\frac{xz}{e^z - xy}}.$$