MATH 2339 , HW 15 : Solution

§11.3

(13)

52/ $u = e^{xy}\sin y$

(3)

$u_x = ye^{xy}\sin y$, $u_y = xe^{xy}\sin y + e^{xy}\cos y$

$\boxed{u_{xy}} = ye^{xy}\cos y + \frac{\partial}{\partial y}(ye^{xy})\sin y = ye^{xy}\cos y + (e^{xy} + yxe^{xy})\sin y$

$\boxed{u_{yx}} = (xye^{xy} + e^{xy})\sin y + ye^{xy}\cos y$

$u_{xy} = u_{yx}$

57/ $u = e^{r\theta}\sin\theta$

(3)

$$\frac{\partial u}{\partial \theta} = re^{r\theta}\sin\theta + e^{r\theta}\cos\theta$$

$$\frac{\partial^2 u}{\partial r\partial\theta} = \frac{\partial}{\partial r}\left(\frac{\partial u}{\partial\theta}\right) = (e^{r\theta} + r\theta e^{r\theta})\sin\theta + \theta e^{r\theta}\cos\theta$$

$$\frac{\partial^3 u}{\partial r^2\partial\theta} = \frac{\partial}{\partial r}\left(\frac{\partial^2 u}{\partial r\partial\theta}\right) = (\theta e^{r\theta} + \theta e^{r\theta} + r\theta^2 e^{r\theta})\sin\theta + \theta^2 e^{r\theta}\cos\theta$$

$$= e^{r\theta}\left[(2\theta + r\theta^2)\sin\theta + \theta^2\cos\theta\right]$$

$$= \boxed{\theta e^{r\theta}\left[(2 + r\theta)\sin\theta + \theta\cos\theta\right]}$$

63/ $u = \dfrac{1}{\sqrt{x^2+y^2+z^2}} = (x^2+y^2+z^2)^{-\frac{1}{2}}$

(4)

$$u_x = -\tfrac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}\cdot(2x) = -x(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$u_{xx} = -\left[(x^2+y^2+z^2)^{-\frac{3}{2}} + x\left(-\tfrac{3}{2}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}(2x)\right]$$

$$= -\left[(x^2+y^2+z^2)^{-\frac{3}{2}} - 3x^2(x^2+y^2+z^2)^{-\frac{5}{2}}\right]$$

$$= \frac{3x^2 - (x^2+y^2+z^2)}{(x^2+y^2+z^2)^{5/2}} = \boxed{\frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{5/2}}}$$

Similarly: $\boxed{u_{yy} = \dfrac{2y^2-x^2-z^2}{(x^2+y^2+z^2)^{5/2}}}$, $\boxed{u_{zz} = \dfrac{2z^2-x^2-y^2}{(x^2+y^2+z^2)^{5/2}}}$

So $u_{xx} + u_{yy} + u_{zz} = \dfrac{2x^2 + 2y^2 + 2z^2 - (x^2 + z^2 + x^2 + y^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)^{5/2}} = 0.$

i.e. $u = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$ is a soln. of $u_{xx} + u_{yy} + u_{zz} = 0$.

75/. ①

$f_x(x,y) = x + 4y, \quad f_{xy} = 4$

$f_y(x,y) = 3x - y, \quad f_{yx} = 3$

f_{xy} and f_{yx} are continuous, but $f_{xy} \neq f_{yx}$

So it is $\boxed{\text{impossible}}$.

82/. ②

$f(x,y) = \sqrt[3]{x^3 + y^3}. \quad f_x(0,0) = 0$

$f(x,0) = \sqrt[3]{x^3 + 0^3} \neq \sqrt[3]{x^3} = x$

$f_x(x,0) = \frac{d}{dx}\left[f(x,0)\right] = \frac{d}{dx}(x) = 1$

$\boxed{f_x(0,0) = 1}$