MATH 2339, HW19: Solution

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§11.4

25/ 4

$z = 5x^2 + y^2 \quad : \quad (1,2) \to (1.05, 2.1)$

$$\boxed{\Delta z = (5\times 1.05^2 + 2.1^2) - (5\times 1^2 + 2^2)} \quad 1$$

$$= 5(1.05^2 - 1^2) + (2.1^2 - 2^2)$$

$$= 5\,(1.05+1)(1.05-1) + (2.1+2)(2.1-2)$$

$$= 5\times 2.05 \times 0.05 + 4.1 \times 0.1$$

$$= 0.05\,(10.25 + 4.1\times 2) = 0.05\times(10.25 + 8.2)$$

$$= 0.05 \times (18.45) = \boxed{0.9225} \quad 1$$

$$\boxed{dz} = f_x(x,y)\,dx + f_y(x,y)\,dy$$

$$= \boxed{f_x(1,2)\,(1.05-1) + f_y(1,2)\,(2.1-1)} \quad 1$$

$f_x = 10x \Rightarrow f_x(1,2) = 10$

$f_y = 2y \Rightarrow f_y(1,2) = 4$

So $dz = 10\times 0.05 + 4\times 0.1 = \boxed{0.9}$ 1

30/ 3

$PV = 8.31T \Rightarrow P = 8.31\frac{T}{V}$

$$\boxed{dP = \frac{\partial P}{\partial T}dT + \frac{\partial P}{\partial V}dV} \quad 1$$

$$\frac{\partial P}{\partial T} = 8.31\frac{1}{V}, \quad \frac{\partial P}{\partial V} = -8.31\frac{T}{V^2}$$

$(T,V) : (310, 12) \to (305, 12.3)$

$$dP = \boxed{\left[\frac{\partial P}{\partial T}(310,12)\right]} \times \boxed{(305-310)} + \boxed{\left[\frac{\partial P}{\partial V}(310,12)\right]} \times \boxed{(12.3-12)} \quad 1$$

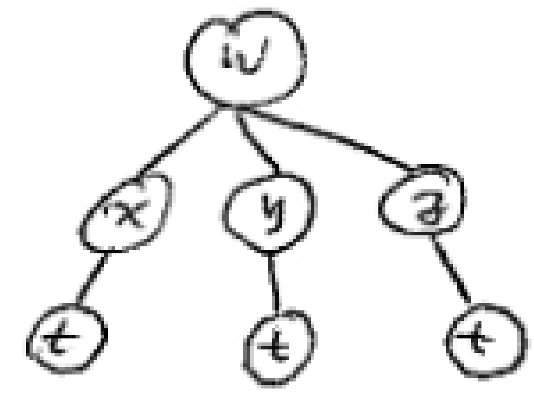
$$= \left[8.31\frac{1}{12}\right]\times(-5) + \left[-8.31\frac{310}{12^2}\right]\times 0.3 = -\frac{8.31\times 153}{144} \boxed{\approx -8.83}$$

1

§11.5

3/. 2

$$w = xe^{y/z}, \quad x = t^2, \; y = 1-t, \; z = 1+2t$$



$$\frac{dw}{dt} = \boxed{\frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}} \; 1$$

$$= \boxed{e^{y/z}\cdot 2t + xe^{y/z}\left(\frac{1}{z}\right)\cdot(-1) + xe^{y/z}\left(-\frac{y}{z^2}\right)\cdot(2)}$$

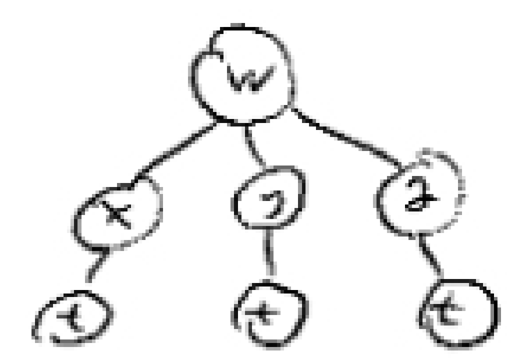
$$= e^{(1-t)/(1+2t)}\cdot 2t + t^2 e^{(1-t)/(1+2t)}\cdot\left(-\frac{1}{1+2t}\right)$$

$$+ t^2 e^{(1-t)/(1+2t)}\left(-2\,\frac{1-t}{(1+2t)^2}\right).$$

$$= \boxed{2te^{(1-t)/(1+2t)} - \frac{t^2}{1+2t}e^{(1-t)/(1+2t)} - 2\,\frac{t^2(1-t)}{(1+2t)^2}\,e^{(1-t)/(1+2t)}} \; 1$$

4/. 2

$$w = \ln\sqrt{x^2+y^2+z^2}, \quad x = \sin t, \; y = \cos t, \; z = \tan t$$



$$\boxed{\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}} \; 1$$

$$\because w = \frac{1}{2}\ln(x^2+y^2+z^2) \Rightarrow$$

$$\frac{\partial w}{\partial x} = \frac{2x}{2(x^2+y^2+z^2)} = \frac{x}{x^2+y^2+z^2}$$

$$\frac{\partial w}{\partial y} = \frac{y}{x^2+y^2+z^2}, \quad \frac{\partial w}{\partial z} = \frac{z}{x^2+y^2+z^2}$$

So $$\frac{dw}{dt} = \left(\frac{x}{x^2+y^2+z^2}\right)\cos t + \left(\frac{y}{x^2+y^2+z^2}\right)(-\sin t)$$

$$+ \left(\frac{z}{x^2+y^2+z^2}\right)\sec^2 t$$

$$= \frac{1}{x^2+y^2+z^2}\left(x\cos t - y\sin t + z\sec^2 t\right)$$

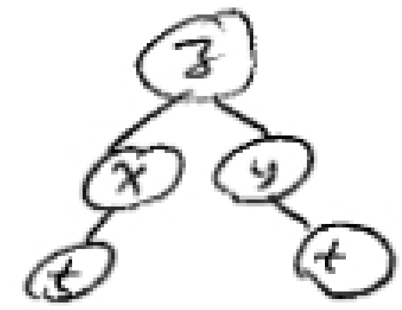
$$= \frac{1}{\sin^2 t + \cos^2 t + \tan^2 t}\left(\sin t\cos t - \cos t\sin t + \tan t\sec^2 t\right)$$

$$= \frac{1}{1+\tan^2 t} \cdot \tan t \sec^2 t = \frac{1}{\sec^2 t} \tan t \sec^2 t = \boxed{\tan t}.$$

1

9/. 2

$z = f(x, y)$, $x = g(t)$, $y = h(t)$



$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$= z_x \cdot g'(t) + z_y \, h'(t)$$

$$\left.\frac{dz}{dt}\right|_{t=3} = z_x\big(\underbrace{g(3)}_{x|_{t=3}}, \underbrace{h(3)}_{y|_{t=3}}\big)\, g'(3) + z_y\big(g(3), h(3)\big)\cdot h'(3)$$

$$= \boxed{f_x(2, 7) \times 5 + f_y(2, 7) \times (-4)}$$ 1

$$= 6 \times 5 + (-8) \times (-4)$$

$$= 30 + 32 = \boxed{62.}$$ 1