

Homework assignment 2 in MATH 309 - Spring 2013 Solutions

Problem 1

$$A = \begin{pmatrix} 2 & -1 & 5 \\ -3 & 4 & 1 \\ 6 & -5 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 3 & -4 \\ 4 & -3 & -1 \\ 2 & -1 & -3 \end{pmatrix}$$

$$(a) \quad 3A - BA = \begin{pmatrix} 6 & -3 & 15 \\ -9 & 12 & 3 \\ 18 & -15 & -12 \end{pmatrix} - \begin{pmatrix} -2 & 3 & -4 \\ 4 & -3 & -1 \\ 2 & -1 & -3 \end{pmatrix} \begin{pmatrix} 2 & -1 & 5 \\ -3 & 4 & 1 \\ 6 & -5 & -4 \end{pmatrix} =$$

$$= \begin{pmatrix} 6 & -3 & 15 \\ -9 & 12 & 3 \\ 18 & -15 & -12 \end{pmatrix} - \begin{pmatrix} -4 - 9 - 24 & 2 + 12 + 20 & -10 + 3 + 16 \\ 8 + 9 - 6 & -4 - 12 + 5 & 20 - 3 + 4 \\ 4 + 3 - 18 & -2 - 4 + 15 & 10 - 1 + 12 \end{pmatrix} =$$

$$= \begin{pmatrix} 6 & -3 & 15 \\ -9 & 12 & 3 \\ 18 & -15 & -12 \end{pmatrix} - \underbrace{\begin{pmatrix} -37 & 34 & 9 \\ 11 & -11 & 21 \\ -11 & 9 & 21 \end{pmatrix}}_{BA} = \begin{pmatrix} 43 & -37 & 6 \\ -20 & 23 & -18 \\ 29 & -24 & -33 \end{pmatrix}$$

$$(b) \quad A^T B^T = (BA)^T = \begin{pmatrix} -37 & 11 & -11 \\ 34 & -11 & 9 \\ 9 & 21 & 21 \end{pmatrix}$$

from the calculation of BA in the previous item

$$(c) \quad (BA)^T = A^T B^T = \text{the same answer as in (b)}$$

(a) Both AB and BA make sense

$$AB = \begin{pmatrix} 1 & 5 & -4 \\ 2 & -7 & 8 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ -2 & 1 \\ 3 & -4 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 \cdot 4 + 5 \cdot (-2) + (-4) \cdot 3 & 1 \cdot (-3) + 5 \cdot (-4) + (-4) \cdot (-4) \\ 2 \cdot 4 + (-7) \cdot (-2) + 8 \cdot 3 & 2 \cdot (-3) + (-7) \cdot (-4) + 8 \cdot (-4) \end{pmatrix} =$$

$$= \begin{pmatrix} 4 - 10 - 12 & -3 + 5 + 16 \\ 8 + 14 + 24 & -6 - 7 - 32 \end{pmatrix} = \begin{pmatrix} -18 & 18 \\ 46 & -45 \end{pmatrix}$$

$$BA = \begin{pmatrix} 4 & -3 \\ -2 & 1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 5 & -4 \\ 2 & -7 & 8 \end{pmatrix} = \begin{pmatrix} 4 - 6 & 20 + 21 & -16 - 24 \\ -2 + 2 & -10 - 7 & 8 + 8 \\ 3 - 8 & 15 + 28 & -12 - 32 \end{pmatrix} =$$

$$= \begin{pmatrix} -2 & 41 & -40 \\ 0 & -17 & 16 \\ -5 & 43 & -44 \end{pmatrix}$$

(b) AB does not make sense because # columns of A = 3 \neq
rows of B = 2

BA makes sense:

$$BA = \begin{pmatrix} 4 & -3 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 5 & -4 \\ 2 & -7 & 8 \end{pmatrix} = \begin{pmatrix} -2 & 41 & -40 \\ 0 & -17 & 16 \end{pmatrix}$$

The first two rows of
the BA from the previous item

(c) Both AB and BA make sense

$$AB = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix} \begin{pmatrix} -3 & 10 & -1 \end{pmatrix} = \begin{pmatrix} 2 \cdot (-3) & 2 \cdot 10 & 2 \cdot (-1) \\ (-1) \cdot (-3) & (-1) \cdot 10 & (-1) \cdot (-1) \\ (-4) \cdot (-3) & (-4) \cdot 10 & (-4) \cdot (-1) \end{pmatrix} =$$

$$= \begin{pmatrix} -6 & 20 & -2 \\ 3 & -10 & 1 \\ 12 & -40 & 4 \end{pmatrix}$$

$$BA = \begin{pmatrix} -3 & 10 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix} = -3 \cdot 2 + 10 \cdot (-1) + (-1) \cdot (-4) =$$

$$= -6 - 10 + 4 = -12$$

Problem 3 $x_1 \begin{pmatrix} 1 \\ -2 \end{pmatrix} + x_2 \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \Rightarrow$

$$\begin{aligned} x_1 + 4x_2 &= 3 & \xrightarrow{2E_1 + E_2} & 8x_2 + 5x_2 = 6 - 4 \Rightarrow 13x_2 = 2 \Rightarrow \\ -2x_1 + 5x_2 &= -4 \end{aligned}$$

$$\boxed{x_2 = \frac{2}{13}} \Rightarrow x_1 = 3 - 4 \cdot \frac{2}{13} =$$
$$\stackrel{E_1}{=} 3 - \frac{8}{13} = \boxed{\frac{31}{13}}$$

$$\Rightarrow \boxed{b = \frac{31}{13} a_1 + \frac{2}{13} a_2}$$