MATH 2339. HW21: Solution

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§11.5

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$$g(r,s) = f(2r-s,\ s^2-4r)$$
$$\equiv f(x(r,s),\ y(r,s)) \ ,\quad \frac{\partial x}{\partial r} = 2,\ \frac{\partial y}{\partial r} = -4,\ \frac{\partial x}{\partial s} = -1,\ \frac{\partial y}{\partial s} = 2s$$

$$g_r = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r}$$

$$= f_x(x,y)\times 2 + f_y(x,y)\times(-4)$$

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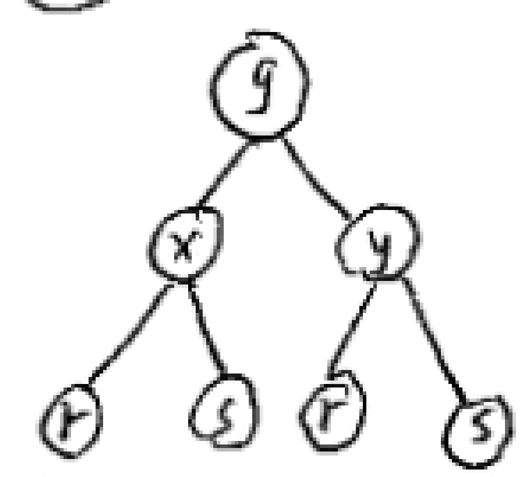

$$r=1,\ s=2 \Rightarrow \begin{cases} x(r,s) = 2\times 1 - 2 = 0 \\ y(r,s) = 2^2 - 4\times 1 = 0 \end{cases}$$

So
$$g_r(1,2) = 2f_x(0,0) - 4f_y(0,0)$$
$$= 2\times 4 - 4\times 8 = \boxed{-24}$$ 1

$$g_s = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$

$$\Rightarrow g_s(1,2) = f_x(0,0)\cdot(-1) + f_y(0,0)(2\times 2)$$ 1

$$= -4 + 4\times 8 = \boxed{28}$$ 1

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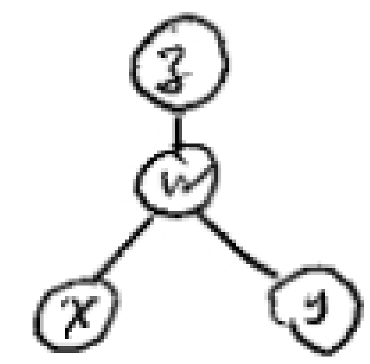
$$z = f(x-y)$$

$\boxed{\text{Let } w = x-y}$ 1 Then $z = f(w(x,y))$



$$\frac{\partial z}{\partial x} = \frac{df}{dw}\frac{\partial w}{\partial x} = \frac{df}{dw}$$

$$\frac{\partial z}{\partial y} = \frac{df}{dw}\frac{\partial w}{\partial y} = -\frac{df}{dw}$$

So $$\boxed{\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = \frac{df}{dw} - \frac{df}{dw} = 0}$$ 1

40/. $z = f(x,y)$, $x = s+t$, $y = s-t$

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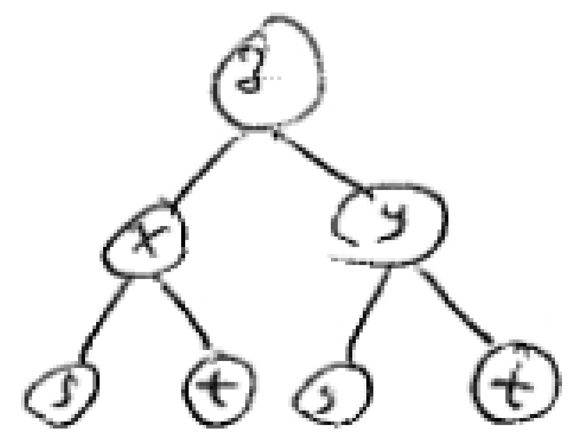

$$\boxed{\frac{\partial z}{\partial s}} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$= \frac{\partial z}{\partial x}(1) + \frac{\partial z}{\partial y}(1) = \boxed{\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}} \ 1$$

$$\boxed{\frac{\partial z}{\partial t}} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$= \frac{\partial z}{\partial x}(1) + \frac{\partial z}{\partial y}(-1) = \boxed{\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}} \ 1$$

So $\frac{\partial z}{\partial s}\frac{\partial z}{\partial t} = \left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right)\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right) \overset{1}{=} \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2.$

43/. $z = f(x,y)$, $x = r^2 + s^2$, $y = 2rs$

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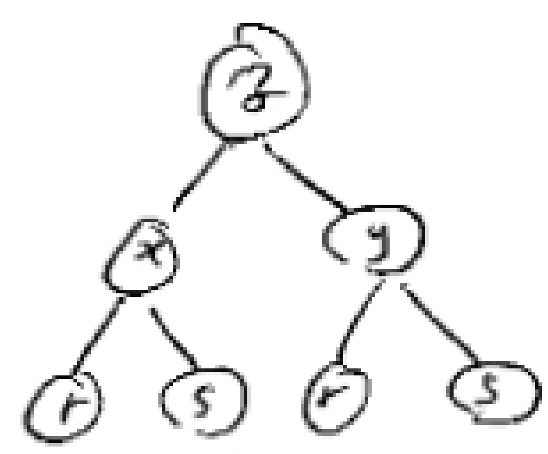
$$\frac{\partial^2 z}{\partial r \partial s} = \frac{\partial}{\partial r}\left(\frac{\partial z}{\partial s}\right)$$



$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

$$= \frac{\partial z}{\partial x}(2s) + \frac{\partial z}{\partial y}(2r)$$

$$\frac{\partial}{\partial r}\left(\frac{\partial z}{\partial s}\right) = \frac{\partial}{\partial r}\left(2s\frac{\partial z}{\partial x}\right) + \frac{\partial}{\partial r}\left(2r\frac{\partial z}{\partial y}\right)$$

$$= 2s\frac{\partial}{\partial r}\left(\frac{\partial z}{\partial x}(x,y)\right) + \left[2r\frac{\partial}{\partial r}\left(\frac{\partial z}{\partial y}(x,y)\right) + 2\frac{\partial z}{\partial y}\right]$$

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$$= 2s\left[\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\frac{\partial x}{\partial r} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\frac{\partial y}{\partial r}\right] + 2r\left[\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\frac{\partial x}{\partial r} + \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\frac{\partial y}{\partial r}\right]$$

$$+ 2\frac{\partial z}{\partial y} = 2s\left(\frac{\partial^2 z}{\partial x^2}(2r) + \frac{\partial^2 z}{\partial y \partial x}(2s)\right) +$$

$$2r\left[\frac{\partial^2 z}{\partial x \partial y}(2r) + \frac{\partial^2 z}{\partial y^2}(2s)\right] + 2\frac{\partial z}{\partial y}$$

$$= 4sr\frac{\partial^2 z}{\partial x^2} + 4s^2\frac{\partial^2 z}{\partial y\partial x} + 4r^2\frac{\partial^2 z}{\partial x\partial y} + 4sr\frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial y}$$

$$= \boxed{4sr\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) + (4s^2+4r^2)\frac{\partial^2 z}{\partial x\partial y} + 2\frac{\partial z}{\partial y}} \quad 1$$

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