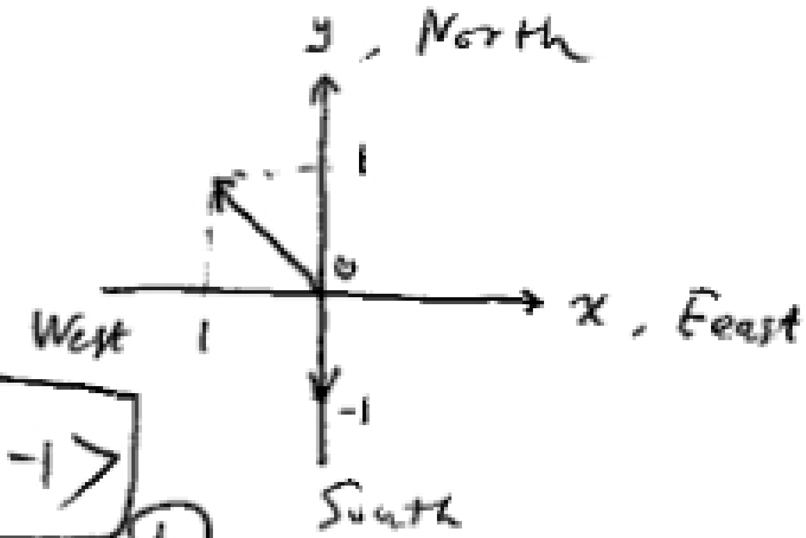
MATH 2339, HW 24: Solution

§11.6

26/ (6)

$z = f(x,y) = 1000 - 0.005x^2 - 0.01y^2$



$P(60, 40, 966)$

$\nabla f = \langle -0.01x, -0.02y \rangle$

(a) To south, the direction $\boxed{\vec{s} = \langle 0, -1 \rangle}$ (1)

$\boxed{\nabla f(60,40)} = \langle f_x(60,40), f_y(60,40) \rangle$

$= \langle -0.01 \times 60, -0.02 \times 40 \rangle$

$= \boxed{\langle -0.6, -0.8 \rangle}$ (1)

So $\boxed{D_{\vec{s}} f(60,40)} = \nabla f(60,40) \cdot \vec{s}$

$= \langle -0.6, -0.8 \rangle \cdot \langle 0, -1 \rangle = \boxed{0.8 > 0}$ (1)

You start to $\boxed{\text{ascend at the rate } 0.8.}$ (1)

(1 meter to south → 0.8 meter above the standing point)

(b) To northwest, the direction is along $\vec{v} = \langle 1, 1 \rangle$

Unit vector $\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{\sqrt{2}}{2} \langle 1, 1 \rangle$

$D_{\vec{u}} f(60,40) = \nabla f(60,40) \cdot \vec{u} = \langle -0.6, -0.8 \rangle \cdot \left(\frac{\sqrt{2}}{2} \langle 1, 1 \rangle \right)$

$= \frac{\sqrt{2}}{2} [(-0.6) + (-0.8)] = -0.7\sqrt{2} < 0$

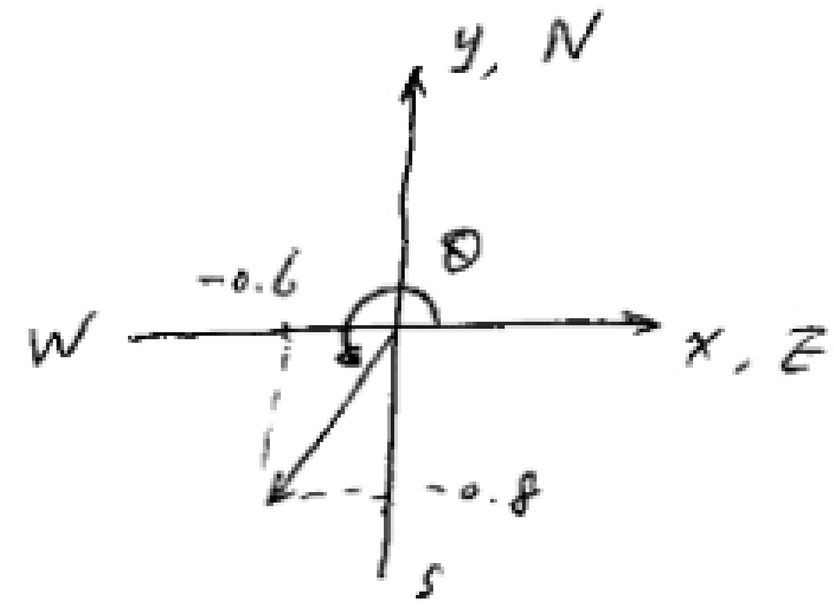
So you start to descend at the rate of $-0.7\sqrt{2}$.

(c) In the direction of $\nabla f(60,40) = \boxed{\langle -0.6, -0.8 \rangle}$ (1)
the slope is largest.

The rate of ascend in this direction is:

$$|\nabla f(60,40)| = \sqrt{(-0.6)^2 + (-0.8)^2} \; \boxed{=1}$$ (1)

The angle of the path above the horizontal is:



$$\theta = \pi + \arctan\left(\frac{0.8}{0.6}\right)$$

$$= \pi + \arctan\left(\frac{4}{3}\right).$$

31/. (4)

$$2(x-2)^2 + (y-1)^2 + (z-3)^2 = 0, \qquad (3,3,5)$$

Let $\boxed{F(x,y,z) = 2(x-2)^2 + (y-1)^2 + (z-3)^2}$ (1)

Then $F(x,y,z) = 0$ is a level surface of F.

(a) The tangent plane to the level surface at $(3,3,5)$:

$$F_x(3,3,5)(x-3) + F_y(3,3,5)(y-3) + F_z(3,3,5)(z-5) = 0$$

$F_x = 4(x-2)$, $F_x(3,3,5) = 4$

$F_y = 2(y-1)$, $F_y(3,3,5) = 4$ (1)

$F_z = 2(z-3)$, $F_z(3,3,5) = 4$

So the plane is:

$$\boxed{4(x-3) + 4(x-3) + 4(z-5) = 0.}$$ (1)

(or: $(x-3) + (y-3) + (z-5) = x + y + z - 11 = 0$)

(b) The normal line is :

$$\frac{x-3}{F_x(3,3,5)} = \frac{y-3}{F_y(3,3,5)} = \frac{z-5}{F_z(3,3,5)}$$

i.e. $\boxed{\frac{x-3}{4} = \frac{y-3}{4} = \frac{z-3}{4}}$ ①

(or $x-3 = y-3 = z-3$)

39/ ③

$f(x,y) = xy$

$\nabla f = \langle f_x, f_y \rangle = \langle y, x \rangle$

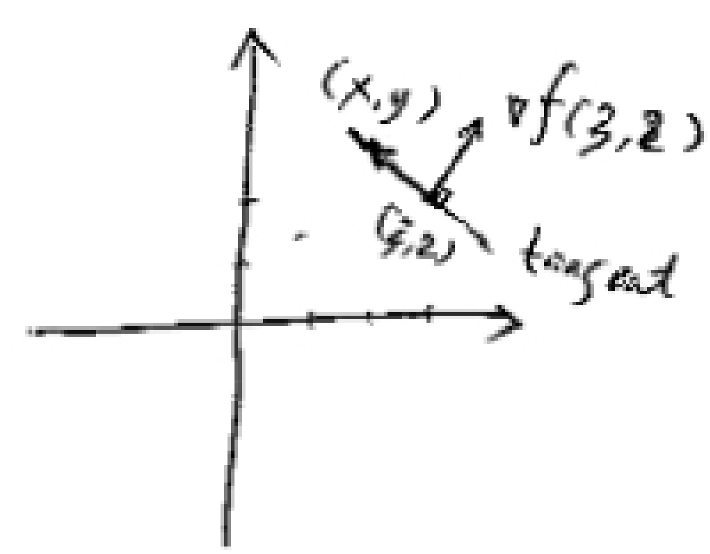
$\nabla f(3,2) = \langle 2, 3 \rangle$ ①

(1) The tangent line to the level curve :

$\nabla f(3,2) \cdot \langle x-3, y-2 \rangle = 0$

i.e. $\boxed{2(x-3) + 3(y-2) = 0}$ ①

(or : $2x+3y-12=0$)

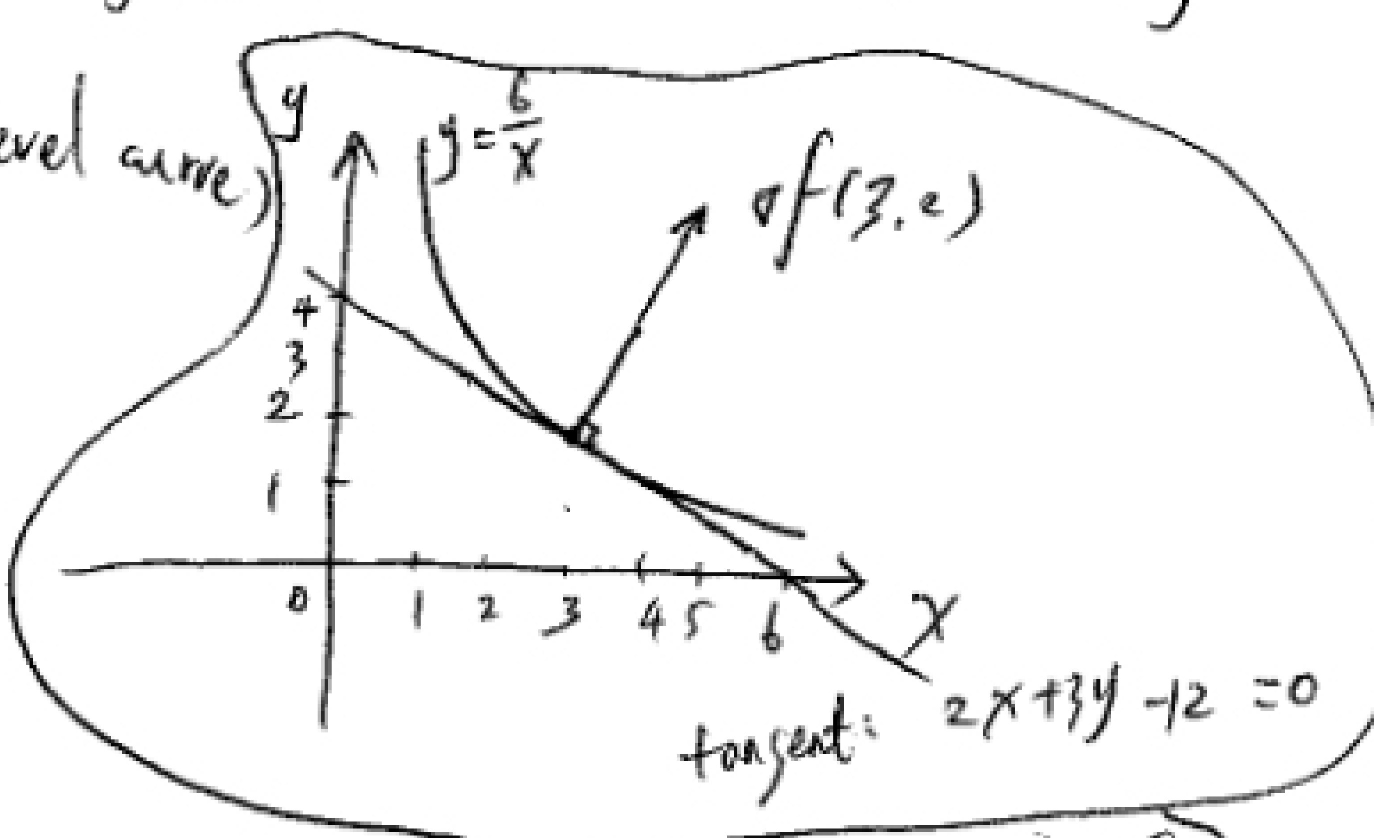


(2) $f(x,y) = xy = f(3,2) = 3\times 2 = 6$, i.e. $xy=6$

$\Rightarrow y = \frac{6}{x}$ (level curve)

$\nabla f(3,2) = \langle 2, 3 \rangle$

$|\nabla f(3,2)| = \sqrt{13}$



②