MATH 2339, HW 25 : Solution (12)

§11.7

(9) (1) @ (x_0, y_0)

* max : moving away from (x_0, y_0) in any direction, the values of level curves decreasing ; ①

* min : moving away from (x_0, y_0) in any direction, the values of level curves increasing ; ①

* Saddle point : moving away from (x_0, y_0), the values of level curves increasing in one direction, but decreasing in another. ①

So :

critical points	classification
(-1, 1)	local min.
(-1, 0)	saddle pt.
(-1, -1)	local min
(1, 1)	saddle pt.
(1, 0)	local max.
(1, -1)	saddle pt.

③

③ (2) Test (Verify) :

$f(x,y) = 3x - x^3 - 2y^2 + y^4$

$f_x(x,y) = 3 - 3x^2 = 3(1-x^2) = 0 \Rightarrow x = 1, -1$

$f_y(x,y) = -4y + 4y^3 = 4y(y^2-1) = 0 \Rightarrow y = 0, 1, -1$

So critical points are : $(1,0), (1,1), (1,-1), (-1,0), (-1,1), (-1,-1)$ //

$f_{xx}(x,y) = -6x$

$f_{yy}(x,y) = -4 + 12y^2$

$f_{xy}(x,y) = 0$

$D = f_{xx} f_{yy} - (f_{xy})^2 = -6x \cdot (-4 + 12y^2)$

Critical pts.	f_{xx}	D	local max/min.
$(-1,1)$	$6 > 0$	$48 > 0$	min
$(-1,0)$	$6 > 0$	$-24 < 0$	Saddle
$(-1,-1)$	$6 > 0$	$48 > 0$	min
$(1,1)$	$-6 < 0$	$-48 < 0$	Saddle
$(1,0)$	$-6 < 0$	$24 > 0$	max
$(1,-1)$	$-6 < 0$	$-48 < 0$	Saddle

3. (3)

$f(x,y) = x^2 + xy + y^2 + y$

$f_x = 2x + y$

$f_y = x + 2y + 1$

$\begin{cases} f_x = 0 : & 2x + y = 0 \\ f_y = 0 & x + 2y + 1 = 0 \end{cases}$ Substitution: $y = -2x \Rightarrow \begin{cases} x = 1/3 \\ y = -2/3 \end{cases}$

The critical pt : $\left(\frac{1}{3}, -\frac{2}{3}\right)$ ①; $f\left(\frac{1}{3}, -\frac{2}{3}\right) = \frac{1}{9} - \frac{2}{9} + \frac{4}{9} - \frac{6}{9} = -\frac{1}{3}$

$f_{xx} = 2$, $f_{yy} = 2$, $f_{xy} = 1$ $\Rightarrow$ $D = f_{xx} f_{yy} - (f_{xy})^2 = 3 > 0$ ①

Since $f_{xx} > 0$, $D > 0$ @ $\left(\frac{1}{3}, -\frac{2}{3}\right)$ local min. at $\left(\frac{1}{3}, -\frac{2}{3}\right)$ ①

local min. value $= -1/3$