MATH 2339, HW28 : Solution

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§12.1

22/. (3) $\iint_R (y + xy^{-2})\, dA, \quad R = \{(x,y) \mid 0 \le x \le 2,\ 1 \le y \le 2\}$

$$\iint_R (y+xy^{-2})\, dA = \int_0^2 \int_1^2 (y + xy^{-2})\, dy\, dx$$

(2)

$$= \int_0^2 \left[\frac{1}{2} y^2 + x \frac{y^{-1}}{(-1)}\right]_{y=1}^{y=2} dx$$

$$= \int_0^2 \left[\frac{1}{2}(2^2 - 1^2) - x(2^{-1} - 1^{-1})\right] dx$$

$$= \left[\frac{3}{2}x - \frac{1}{2}x^2\left(-\frac{1}{2}\right)\right]_0^2 = \frac{3}{2}(2-0) + \frac{1}{4}(2^2 - 0^2)$$

$$= 3 + 1 = \boxed{4} \quad (1)$$

23/. (3) $\iint_R x\sin(x+y)\, dA, \quad R = [0, \frac{\pi}{6}] \times [0, \frac{\pi}{3}]$

$$\iint_R x\sin(x+y)\, dA = \int_0^{\frac{\pi}{6}} \int_0^{\frac{\pi}{3}} x\sin(x+y)\, dy\, dx$$

$$= \int_0^{\frac{\pi}{6}} \left\{x\left[-\cos(x+y)\right]\right\}_{y=0}^{y=\frac{\pi}{3}} dx$$

(2)

$$= \int_0^{\frac{\pi}{6}} x\left[-\cos\left(x+\frac{\pi}{3}\right) + \cos x\right] dx$$

$$= \int_0^{\frac{\pi}{6}} x\cos x\, dx - \int_0^{\frac{\pi}{6}} x\cos\left(x+\frac{\pi}{3}\right) dx$$

$$= \left[x\sin x\right]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \sin x\, dx - \left\{\left[x\sin\left(x+\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} \sin\left(x+\frac{\pi}{3}\right) dx\right\}$$

$$= \frac{\pi}{6}\sin\frac{\pi}{6} + \left[\cos x\right]_0^{\frac{\pi}{6}} - \frac{\pi}{6}\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right) - \left[\cos\left(x+\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{12} + \left(\frac{\sqrt{3}}{2} - 1\right) - \frac{\pi}{6} - \left(0 - \frac{1}{2}\right) = \boxed{\frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{\pi}{12}} \quad 1$$

29. 4

$$4x + 6y - 2z + 15 = 0 \Rightarrow z = 2x + 3y + \frac{15}{2}$$

$$\boxed{V = \iint_R \left(2x + 3y + \frac{15}{2}\right) dA} \quad 1$$

$$= \int_{-1}^{2} \int_{-1}^{1} \left(2x + 3y + \frac{15}{2}\right) dy\, dx$$

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$$= \int_{-1}^{2} \left[2xy + \frac{3}{2}y^2 + \frac{15}{2}y\right]_{y=-1}^{y=1} dx$$

$$= \int_{-1}^{2} \left\{2x[1-(-1)] + \frac{3}{2}\left(1^2 - (-1)^2\right) + \frac{15}{2}(1-(-1))\right\} dx$$

$$= \int_{-1}^{2} (4x + 15)\, dx = \left[4\frac{x^2}{2} + 15x\right]_{-1}^{2}$$

$$= \frac{4}{2}\left[2^2 - (-1)^2\right] + 15\left[2 - (-1)\right] = 6 + 45 = \boxed{51} \quad 1$$

40. 3

$$\iint_R f(x,y)\, dA, \quad R = [0,4] \times [0,1]$$

$$A(R) = (4-0) \times (1-0) = 4$$

$$\iint_R f(x,y)\, dA = \int_0^4 \int_0^1 e^y \sqrt{x + e^y}\, dy\, dx$$

Let $u = e^y$, then $du = e^y dy$; $e^0 = 1$, $e^1 = e$

$$= \int_0^4 \int_1^e \sqrt{x+u}\, du\, dx$$

$$= \int_0^4 \left[\frac{2}{3}(x+u)^{\frac{3}{2}}\right]_{u=1}^{u=e} dx = \int_0^4 \frac{2}{3}\left[(x+e)^{\frac{3}{2}} - (x+1)^{\frac{3}{2}}\right] dx$$

$$= \frac{2}{3}\left[\frac{2}{5}(x+e)^{\frac{5}{2}} - \frac{2}{5}(x+1)^{\frac{5}{2}}\right]_0^4 = \frac{4}{15}\left[(4+e)^{\frac{5}{2}} - e^{\frac{5}{2}}\right] - \frac{4}{15}\left[5^{\frac{5}{2}} - 1\right]$$

So $$f_{avg} = \frac{\iint_R f(x,y)\,dA}{A(R)} = \frac{1}{4}\,\frac{4}{15}\left[(4+e)^{\frac{5}{2}} - e^{\frac{5}{2}}\right] - \frac{1}{4}\,\frac{4}{15}\left(5^{\frac{5}{2}}-1\right)$$

$$= \frac{1}{15}\left[(4+e)^{\frac{5}{2}} - e^{\frac{5}{2}} - 5^{\frac{5}{2}} + 1\right].$$

41/ (2)

$$\iint_R k\,dA = \int_a^b \int_c^d k\,dy\,dx$$

$$= \int_c^b \left[ky\right]_{y=c}^{y=d}\,dx$$

$$= \int_a^b k(d-c)\,dx$$

$$= \left[k(d-c)x\right]_a^b = k(d-c)(b-a).$$