

§12.2

④ 7/.

$$\iint_D y^2 dA, \quad D = \{(x, y) \mid -1 \leq y \leq 1, -y-2 \leq x \leq y\}$$

$$\iint_D y^2 dA =$$

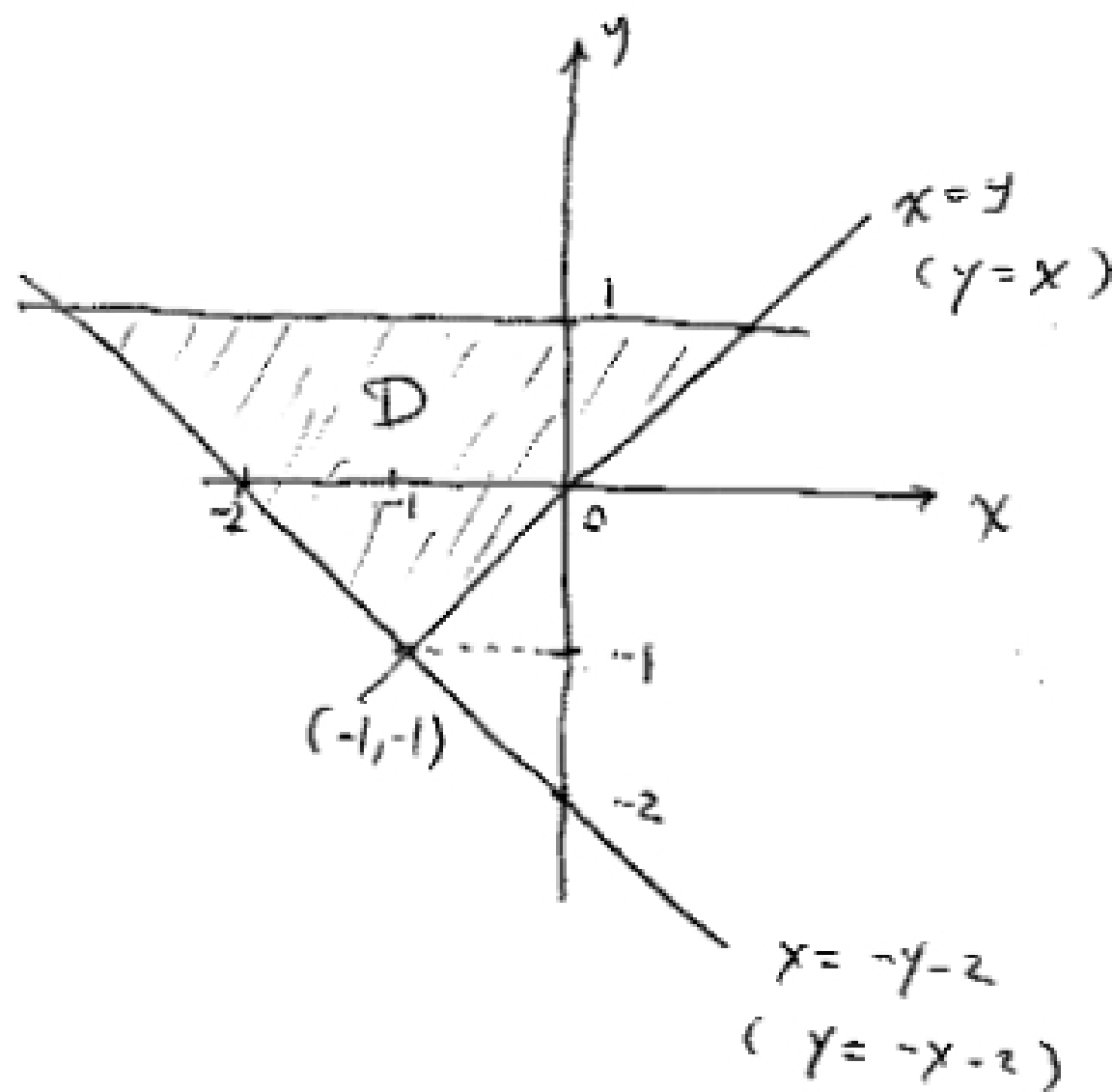
$$\boxed{\int_{-1}^1 \int_{-y-2}^y y^2 dx dy} \quad \text{①}$$

$$= \int_{-1}^1 [y^2 x]_{x=-y-2}^{x=y} dy$$

$$= \int_{-1}^1 y^2 [y - (-y-2)] dy$$

$$= \int_{-1}^1 y^2 (2y+2) dy = \int_{-1}^1 (2y^3 + 2y^2) dy$$

$$= \left[2 \frac{y^4}{4} + 2 \frac{y^3}{3} \right]_{-1}^1 = \frac{1}{2} [1^4 - (-1)^4] + \frac{2}{3} [1^3 - (-1)^3] = \boxed{\frac{4}{3}} \quad \text{①}$$

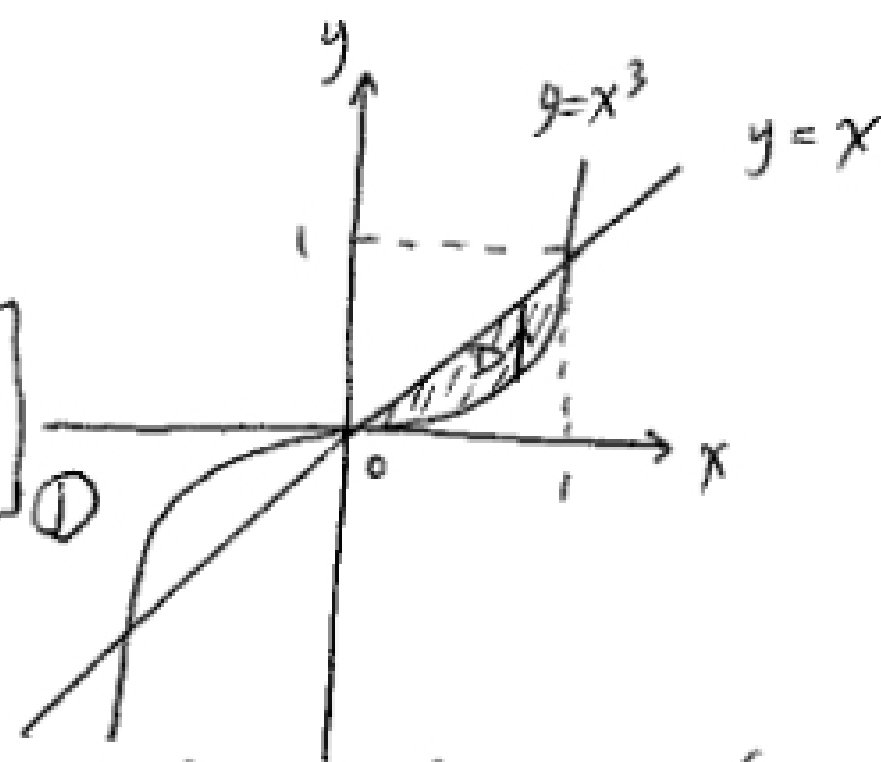


② ↓

16/
⑤

$$\iint_D (x^2 + 2y) dA$$

$$D = \{(x, y) \mid 0 \leq x \leq 1, x^3 \leq y \leq x\}$$



$$\begin{cases} y = x^3 \\ y = x \end{cases} \Rightarrow x^3 = x \Rightarrow x(x^2 - 1) = 0 \Rightarrow x = 0, 1, -1$$

$$S_0: \iint_D (x^2 + 2y) dA$$

$$= \int_0^1 \int_{x^3}^x (x^2 + 2y) dy dx \quad ①$$

$$= \int_0^1 [x^2 y + y^2]_{y=x^3}^{y=x} dx$$

$$= \int_0^1 [x^2 (x - x^3) + (x^2 - (x^3)^2)] dx$$

$$= \int_0^1 (x^3 - x^5 + x^2 - x^6) dx$$

$$= \left[\frac{1}{4} x^4 - \frac{x^5}{5} + \frac{x^3}{3} - \frac{x^7}{7} \right]_0^1$$

$$= \frac{1}{4} - \frac{1}{5} + \frac{1}{3} - \frac{1}{7} = \frac{21 - 14 + 28 - 12}{84} = \boxed{\frac{23}{84}} \quad ①$$

②