

§12.2

④ ^{27/}

$$V = \iint_D xy \, dA$$

$$= \int_1^4 \int_1^{\frac{7}{3} - \frac{1}{3}x} xy \, dy \, dx \quad (1)$$

$$= \int_1^4 \left[x \frac{y^2}{2} \right]_{y=1}^{y=\frac{7}{3} - \frac{1}{3}x} dx$$

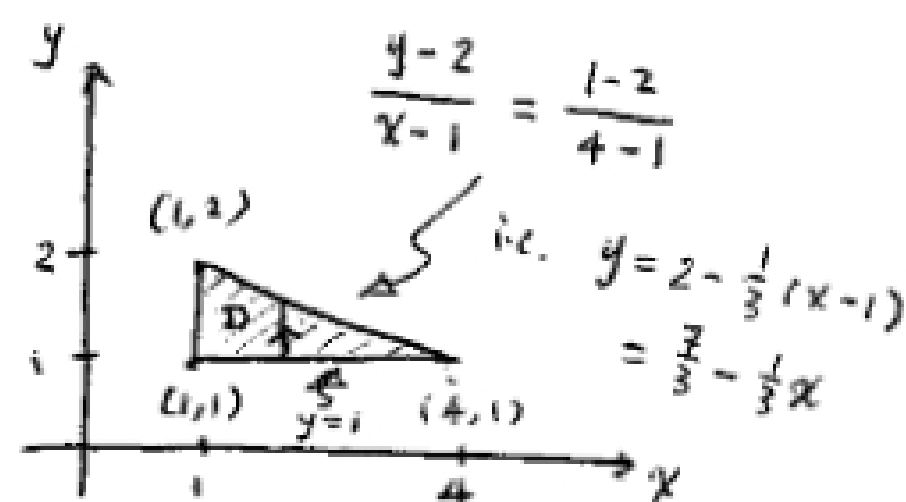
②

$$= \int_1^4 \frac{x}{2} \left(\left[\frac{1}{3}(7-x) \right]^2 - 1^2 \right) dx = \int_1^4 \frac{x}{2} \left[\frac{1}{9}(49 - 14x + x^2) - 1 \right] dx$$

$$= \frac{1}{18} \int_1^4 x(49 - 14x + x^2 - 9) dx = \frac{1}{18} \int_1^4 (40x - 14x^2 + x^3) dx$$

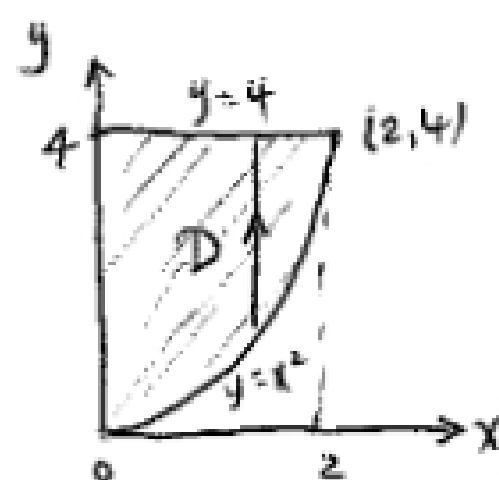
$$= \frac{1}{18} \left[20x^2 - \frac{14}{3}x^3 + \frac{1}{4}x^4 \right]_1^4 = \frac{1}{18} \left[20(4^2 - 1^2) - \frac{14}{3}(4^3 - 1^3) + \frac{1}{4}(4^4 - 1^4) \right]$$

$$= \frac{1}{18} \left(20 \times 15 - \frac{14}{3} \times 63 + \frac{255}{4} \right) = \frac{1}{18} \left(\frac{255}{4} + 6 \right) = \frac{1}{18} \cdot \frac{279}{4} = \boxed{\frac{31}{8}} \quad (1)$$

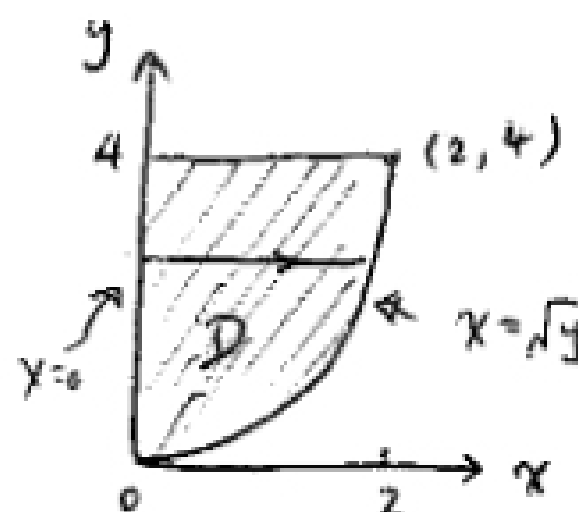


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$$\int_0^2 \int_{x^2}^4 f(x,y) \, dy \, dx$$



or:

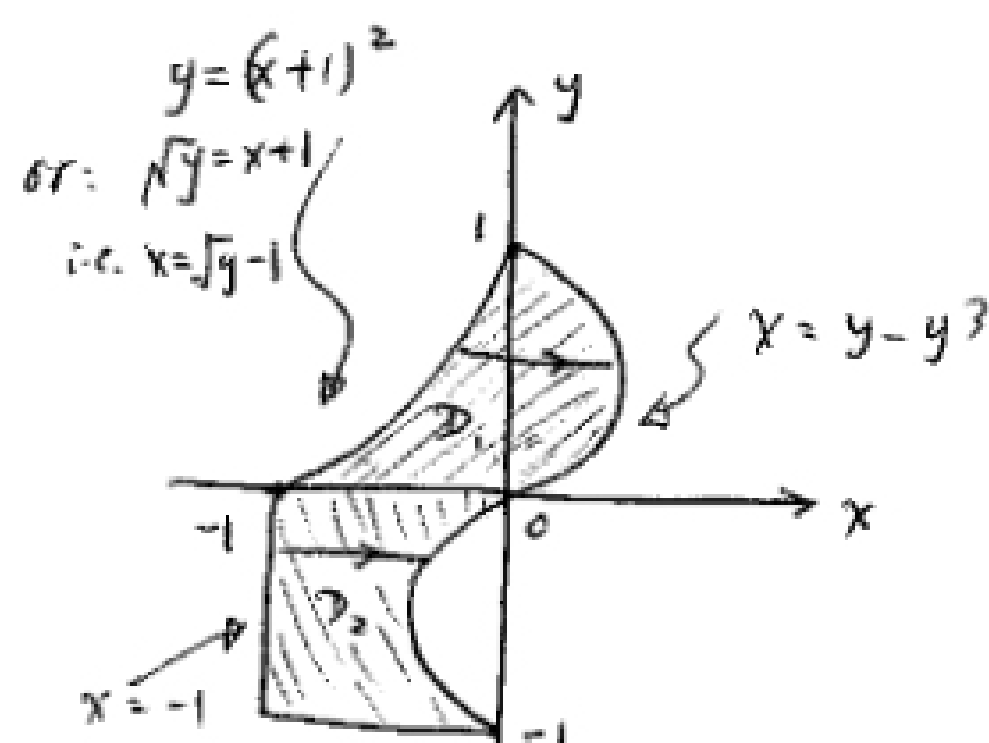


$$\int_0^2 \int_{x^2}^4 f(x,y) \, dy \, dx = \int_0^4 \int_0^{\sqrt{y}} f(x,y) \, dx \, dy$$

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⑥

$$\iint_D y \, dA$$

$$\iint_D y \, dA = \iint_{D_1} y \, dA + \iint_{D_2} y \, dA$$



$$= \int_0^1 \int_{\sqrt{y}-1}^{y-y^3} y \, dx \, dy + \int_{-1}^0 \int_{-1}^{y-y^3} y \, dx \, dy \quad (2)$$

$$= \int_0^1 [yx]_{x=\sqrt{y}-1}^{x=y-y^3} dy + \int_{-1}^0 [yx]_{x=-1}^{x=y-y^3} dy$$

$$= \int_0^1 y[(y-y^3) - (\sqrt{y}-1)] dy + \int_{-1}^0 y[(y-y^3) - (-1)] dy$$

$$= \int_0^1 (y^2 - y^4 - y^{\frac{3}{2}} + y) dy + \int_{-1}^0 (y^2 - y^4 + y) dy$$

$$= \int_{-1}^1 (y^2 - y^4 + y) dy + \int_0^1 (-y^{\frac{3}{2}}) dy$$

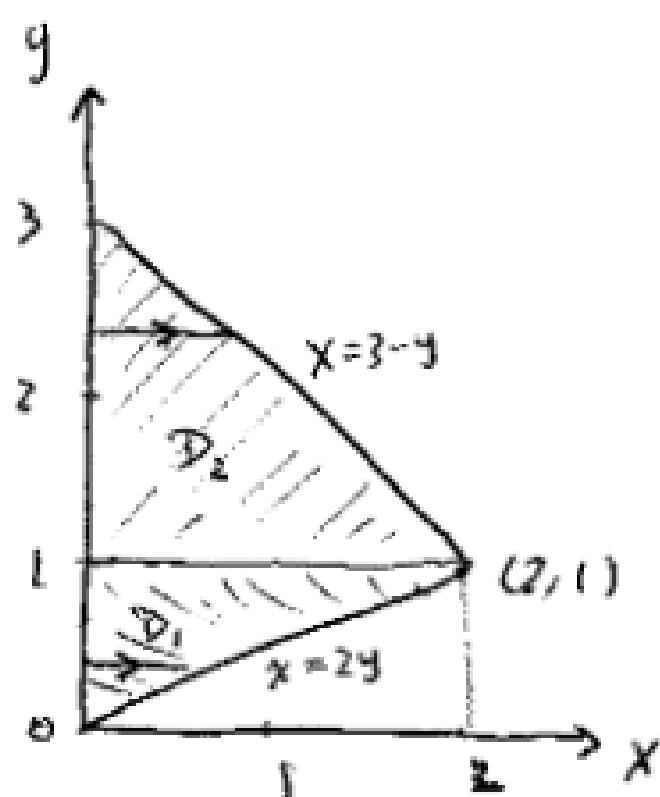
$$= \left[\frac{y^3}{3} - \frac{y^5}{5} + \frac{y^2}{2} \right]_{-1}^1 - \left[\frac{2}{5} y^{\frac{5}{2}} \right]_0^1$$

$$= \frac{2}{3} - \frac{2}{5} - \frac{2}{5} = \frac{2}{3} - \frac{4}{5} = \boxed{\frac{-2}{15}} \quad (1)$$

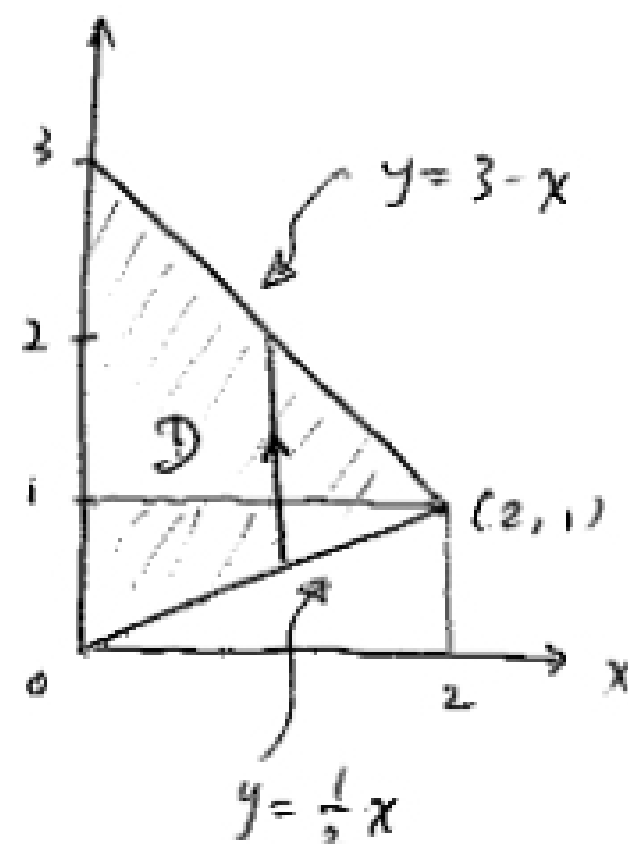
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②

$$\iint_D f(x,y) dA = \int_0^1 \int_0^{2-y} f(x,y) dx dy + \int_1^3 \int_0^{3-y} f(x,y) dx dy$$



or:



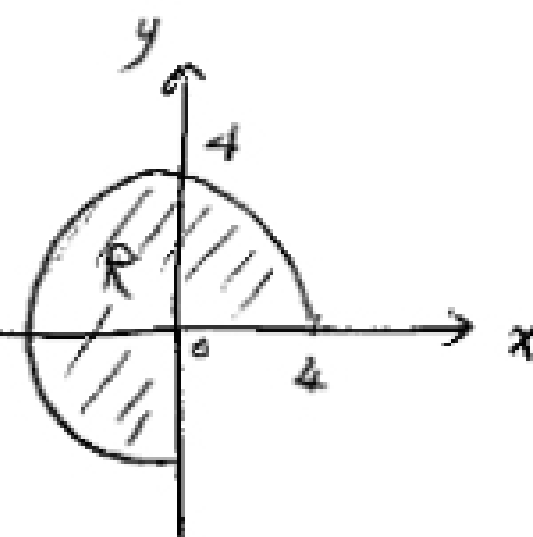
$$\text{So } \iint_D f(x,y) dA = \int_0^2 \int_{\frac{1}{2}x}^{3-x} f(x,y) dy dx$$

Ex 12.3

①

Use polar coordinates:

$$\iint_R f(x,y) dA = \int_0^{\frac{\pi}{2}} \int_0^4 f(r \cos \theta, r \sin \theta) r dr d\theta$$



④

Use polar coordinates

$$\iint_R f(x,y) dA = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_3^6 f(r \cos \theta, r \sin \theta) r dr d\theta$$

