MATH 2339, HW31: Solution

§12.2

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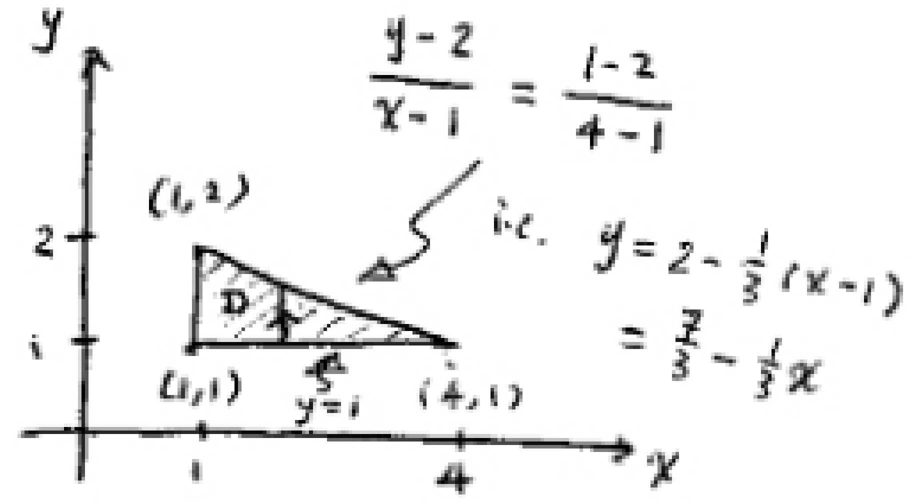
$$V = \iint_D xy\, dA$$



$$= \boxed{\int_1^4 \int_1^{\frac{7}{3}-\frac{1}{3}x} xy\, dy\, dx} \quad 1$$

$$= \int_1^4 \left[x \frac{y^2}{2}\right]_{y=1}^{y=\frac{7}{3}-\frac{1}{3}x} dx$$

2

$$= \int_1^4 \frac{x}{2}\left(\left[\frac{1}{3}(7-x)\right]^2 - 1^2\right) dx = \int_1^4 \frac{x}{2}\left[\frac{1}{9}(49-14x+x^2) - 1\right] dx$$

$$= \frac{1}{18}\int_1^4 x(49-14x+x^2-9)\, dx = \frac{1}{18}\int_1^4 (40x - 14x^2 + x^3)\, dx$$

$$= \frac{1}{18}\left[20x^2 - \frac{14}{3}x^3 + \frac{1}{4}x^4\right]_1^4 = \frac{1}{18}\left[20(4^2-1^2) - \frac{14}{3}(4^3-1^3) + \frac{1}{4}(4^4-1^4)\right]$$

$$= \frac{1}{18}\left(20\times 15 - \frac{14}{3}\times 63 + \frac{255}{4}\right) = \frac{1}{18}\left(\frac{255}{4} + 6\right) = \frac{1}{18}\cdot\frac{279}{4} = \boxed{\frac{31}{8}} \quad 1$$

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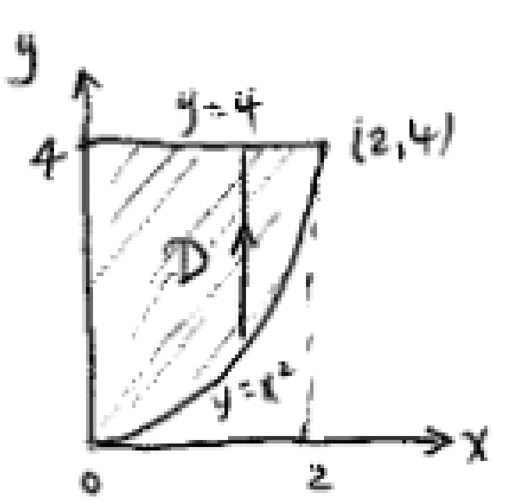
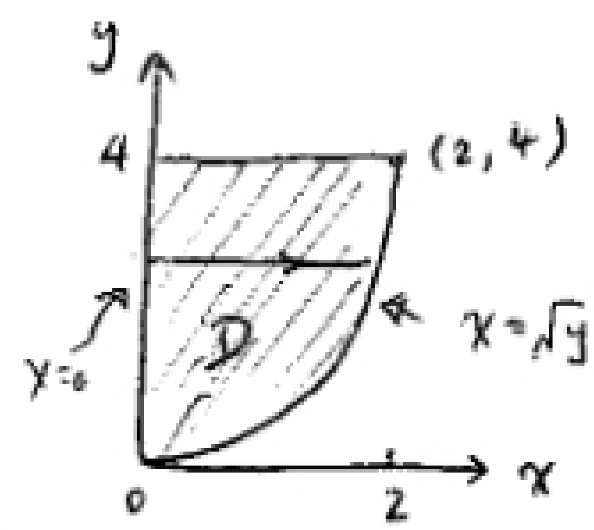
$$\int_0^2 \int_{x^2}^4 f(x,y)\, dy\, dx$$

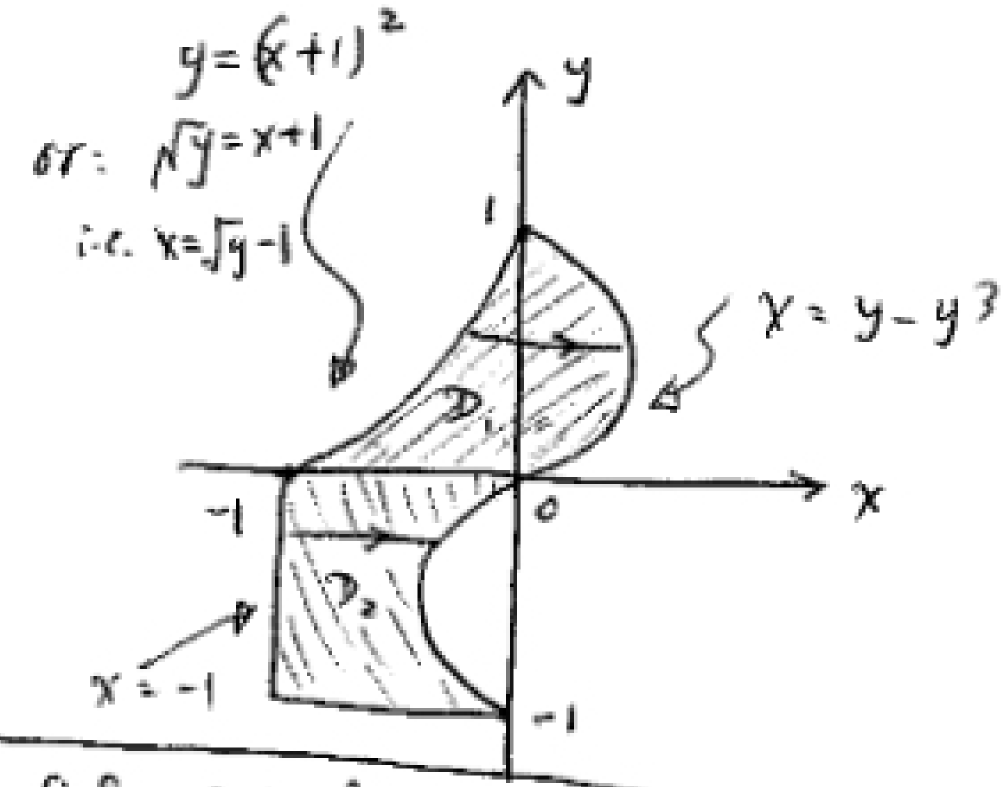


or :



$$\int_0^2 \int_{x^2}^4 f(x,y)\, dy\, dx = \boxed{\int_0^4 \int_0^{\sqrt{y}} f(x,y)\, dx\, dy}$$

50/. 6 $\displaystyle\iint_D y \; dA$



$$\iint_D y\,dA = \iint_{D_1} y\,dA + \iint_{D_2} y\,dA$$

$$= \boxed{\int_0^1 \int_{\sqrt{y}-1}^{y-y^3} y\,dx\,dy \quad + \int_{-1}^{0} \int_{-1}^{y-y^3} y\,dx\,dy} \quad 2$$

$$= \int_0^1 \left[yx\right]_{x=\sqrt{y}-1}^{x=y-y^3} dy + \int_{-1}^{0} \left[yx\right]_{x=-1}^{x=y-y^3} dy$$

3

$$= \int_0^1 y\left[(y-y^3) - (\sqrt{y}-1)\right] dy + \int_{-1}^{0} y\left[(y-y^3) - (-1)\right] dy$$

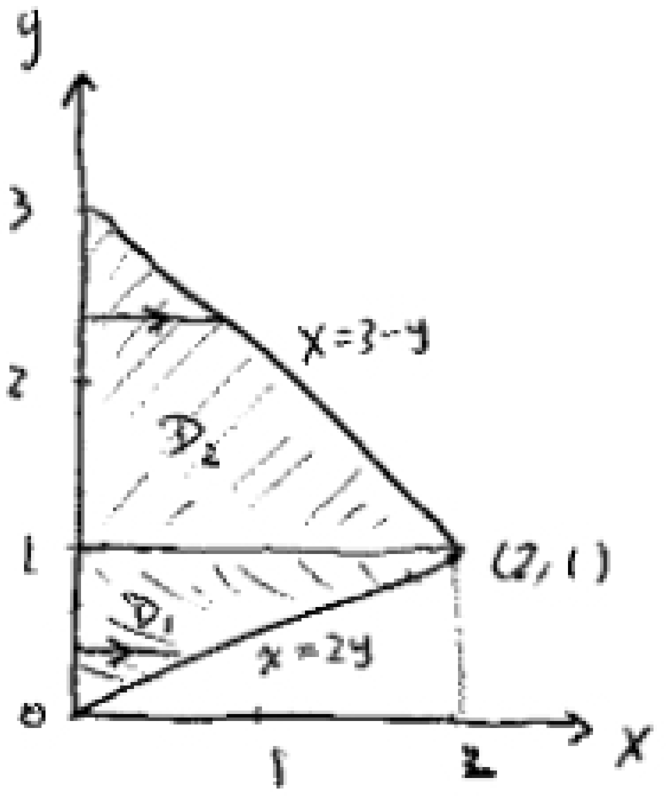
$$= \int_0^1 \left(y^2 - y^4 - y^{\frac{3}{2}} + y\right) dy + \int_{-1}^{0} \left(y^2 - y^4 + y\right) dy$$

$$= \int_{-1}^{1} \left(y^2 - y^4 + y\right) dy + \int_0^1 \left(-y^{\frac{3}{2}}\right) dy$$

$$= \left[\frac{y^3}{3} - \frac{y^5}{5} + \frac{y^2}{2}\right]_{-1}^{1} - \left[\frac{2}{5} y^{\frac{5}{2}}\right]_0^1$$

$$= \frac{2}{3} - \frac{2}{5} - \frac{2}{5} = \frac{2}{3} - \frac{4}{5} = \boxed{\frac{-2}{15}} \quad 1$$

54/.
(2) $$\iint_D f(x,y)\,dA = \int_0^1 \int_0^{2y} f(x,y)\,dx\,dy + \int_1^3 \int_0^{3-y} f(x,y)\,dx\,dy$$



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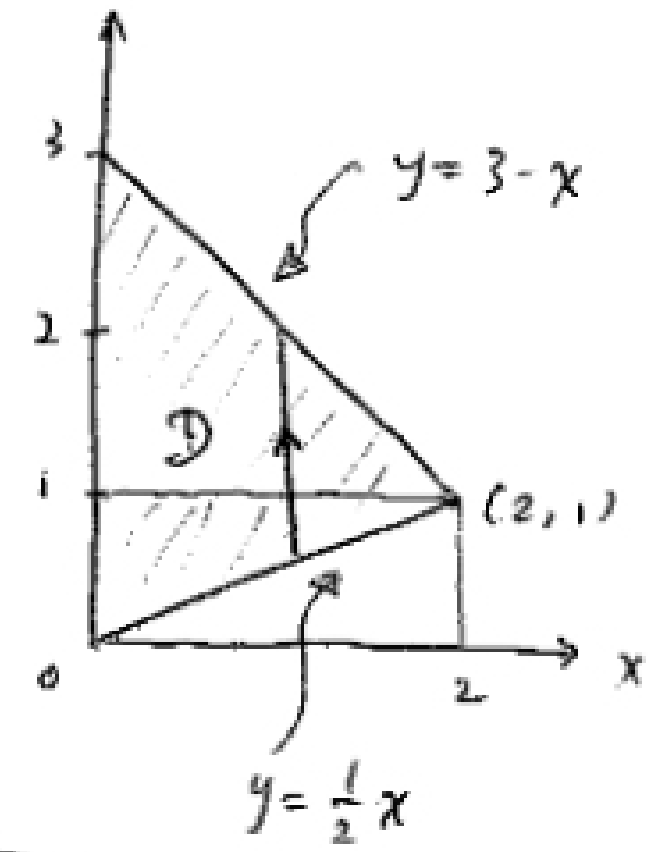
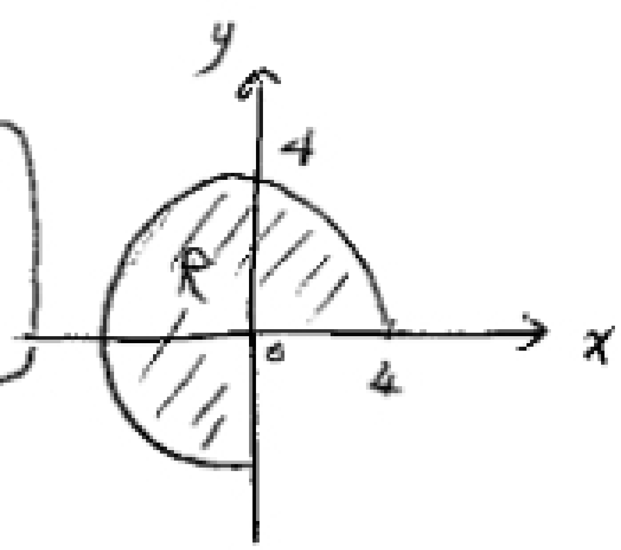


So $$\iint_D f(x,y)\,dA = \boxed{\int_0^2 \int_{\frac{1}{2}x}^{3-x} f(x,y)\,dy\,dx}$$

§12.3

1/ (1) the polar coordinates :

$$\iint_R f(x,y)\,dA = \boxed{\int_0^{\frac{3\pi}{2}} \int_0^4 f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta}$$ (1)



4/. (1) Use polar coordinates

$$\iint_R f(x,y)\,dA = \boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_3^6 f(r\cos\theta, r\sin\theta)\, r\,dr\,d\theta}$$ (1)