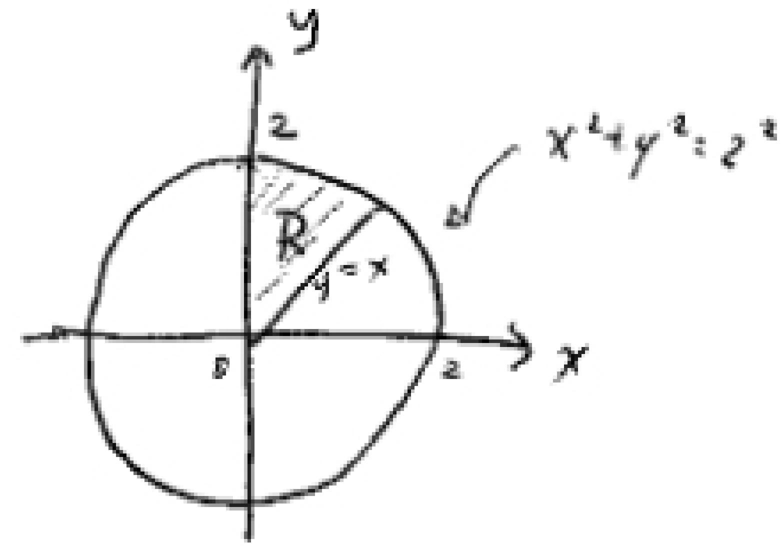
MATH 2339, <u>HW32: Solution</u> 13

§ 12.3

4 8/. $\iint_R (2x-y)\, dA =$




$$\boxed{\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^2 (2r\cos\theta - r\sin\theta)\, r\, dr\, d\theta}$$ 1

2

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^2 (2r^2\cos\theta - r^2\sin\theta)\, dr\, d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[\frac{2r^3}{3}\cos\theta - \frac{r^3}{3}\sin\theta\right]_{r=0}^{r=2} d\theta$$

$$= \int_{\pi/4}^{\pi/2} \left(\frac{2\times 2^3}{3}\cos\theta - \frac{2^3}{3}\sin\theta\right) d\theta = \left[\frac{16}{3}\sin\theta + \frac{8}{3}\cos\theta\right]_{\pi/4}^{\pi/2}$$

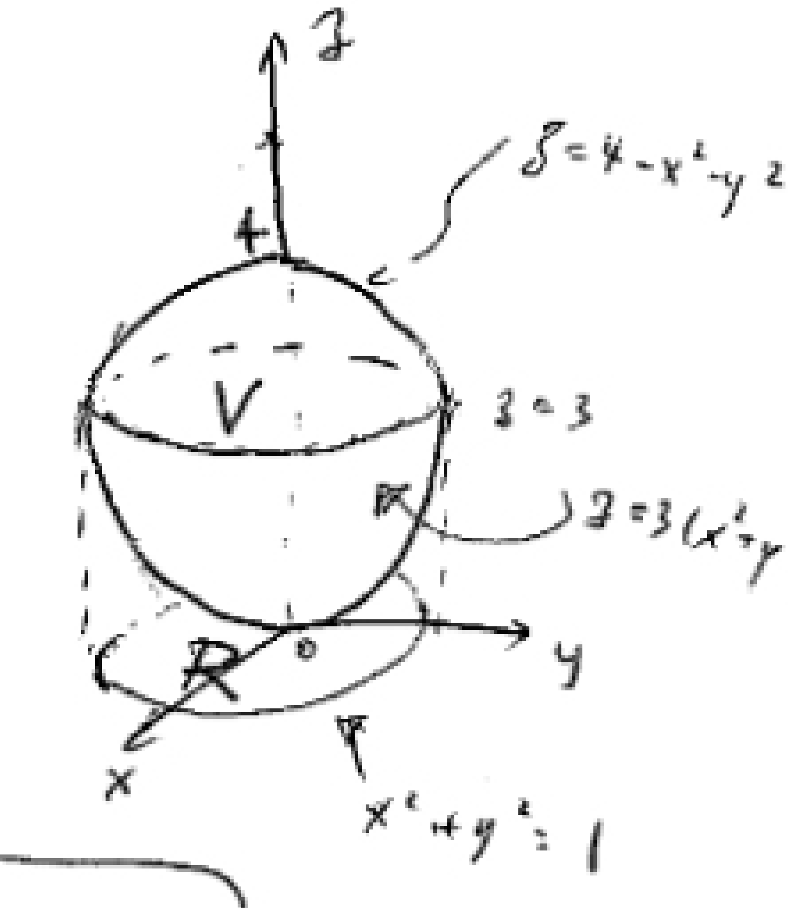
$$= \frac{16}{3}\left(1 - \frac{\sqrt{2}}{2}\right) + \frac{8}{3}\left(0 - \frac{\sqrt{2}}{2}\right) = \frac{16}{3} - \frac{\sqrt{2}}{2}\left(\frac{16}{3} + \frac{8}{3}\right) = \boxed{\frac{16}{3} - 4\sqrt{2}}$$ 1

3 18/.

$$\left.\begin{array}{l} z = 3x^2 + 3y^2 \\ z = 4 - x^2 - y^2 \end{array}\right\} \Rightarrow 3x^2 + 3y^2 = 4 - x^2 - y^2$$

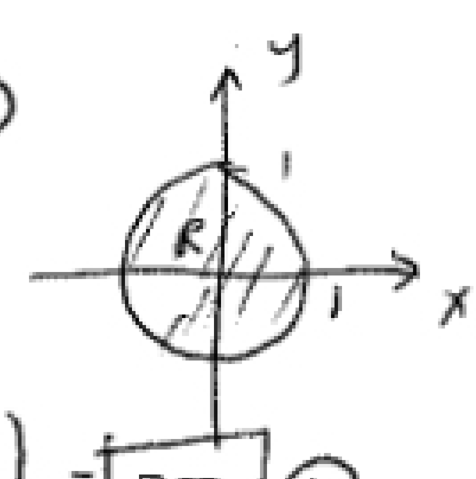
$$\Rightarrow x^2 + y^2 = 1 \ \ \& \ \ z = 3x^2 + 3y^2 = 3$$

So the two surfaces intersect into the circle $x^2+y^2=1$ at $z=3$.




The volume: $$\boxed{V = \iint_R \Big[\underbrace{(4-x^2-y^2) - (3x^2+3y^2)}_{=4-4(x^2+y^2)}\Big]\, dR}$$ 1

$$= \boxed{\int_0^{2\pi} \int_0^1 \Big[\underbrace{4 - 4(r^2\cos^2\theta + r^2\sin^2\theta)}_{=4(1-r^2)}\Big]\, r\, dr\, d\theta}$$ 1




$$= \int_0^{2\pi} \int_0^1 4(1-r^2)\, r\, dr\, d\theta = 8\pi \int_0^1 (r - r^3)\, dr = 8\pi\left(\frac{1}{2} - \frac{1}{4}\right) = \boxed{2\pi}$$ 1

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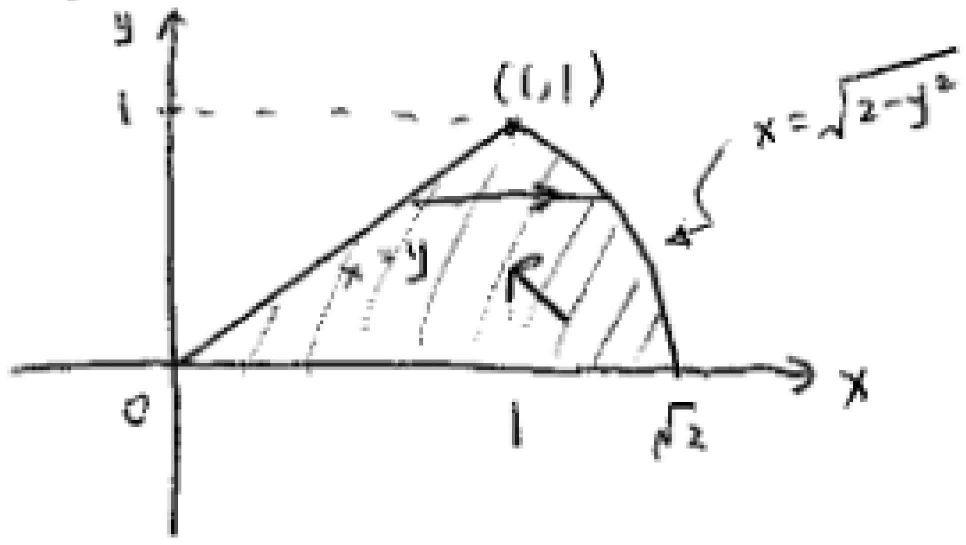
$$\int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)\, dx\, dy$$

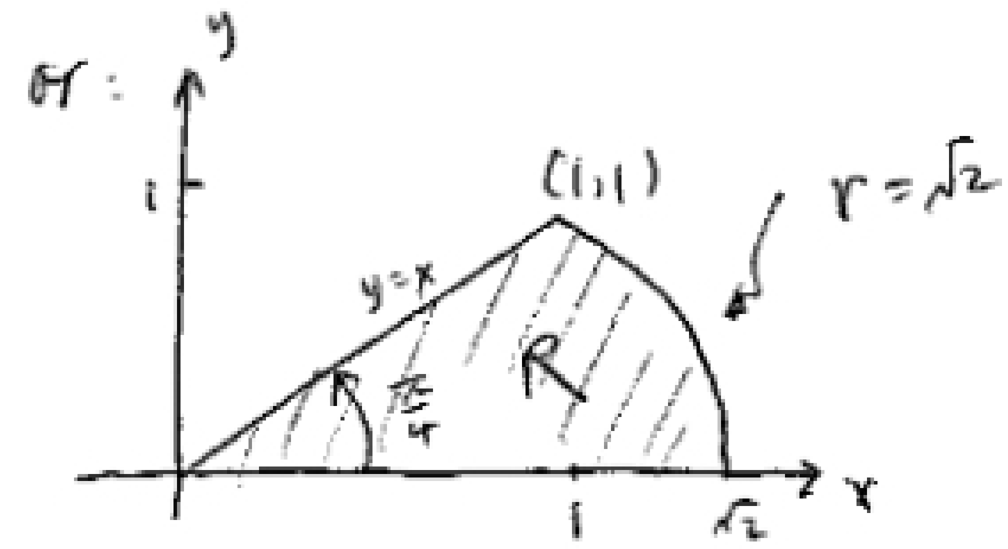



$: \quad r\cos\theta = \sqrt{2 - r^2\sin^2\theta}$

$\Rightarrow r^2\cos^2\theta = 2 - r^2\sin^2\theta$

$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$

or:




$$\text{So}: \int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)\, dx\, dy = \boxed{\int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2}} \underbrace{(r\cos\theta + r\sin\theta)\, r}_{= r^3(\cos\theta+\sin\theta)}\, dr\, d\theta}$$

1

1

$$= \int_0^{\frac{\pi}{4}} \left[\frac{r^3}{3}(\cos\theta + \sin\theta)\right]_{r=0}^{\sqrt{2}} d\theta = \frac{\sqrt{2}^3}{3}\int_0^{\frac{\pi}{4}} (\cos\theta + \sin\theta)\, d\theta$$

$$= \frac{2\sqrt{2}}{3}\left[\sin\theta - \cos\theta\right]_0^{\frac{\pi}{4}} = \frac{2\sqrt{2}}{3}\left[\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) - (0-1)\right] = \boxed{\frac{2\sqrt{2}}{3}}$$

1

(a) $\iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\, dA =$

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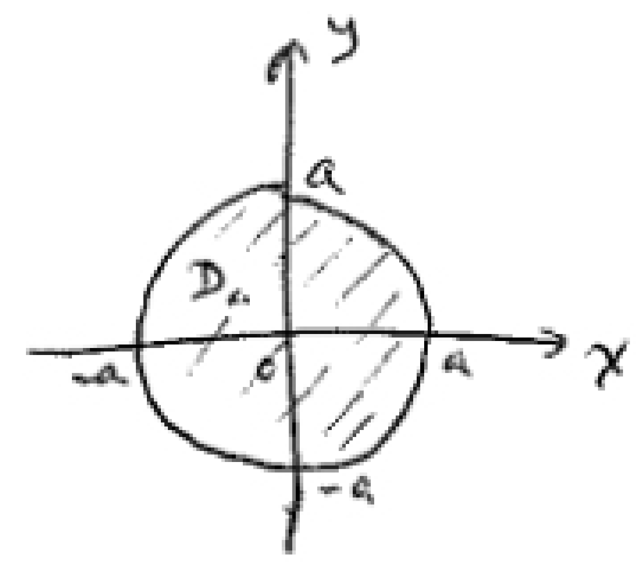
$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, dA =$




$\lim_{a\to\infty} \iint_{D_a} e^{-(x^2+y^2)}\, dA =$

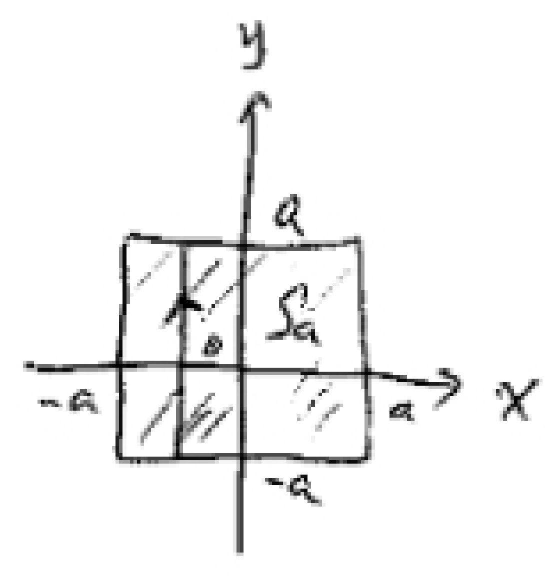
$\lim_{a\to\infty} \int_0^{2\pi}\int_0^{a} e^{-(r^2\cos^2\theta + r^2\sin^2\theta)}\, r\, dr\, d\theta =$

$\lim_{a\to\infty} \int_0^{2\pi}\int_0^{a} re^{-r^2}\, dr\, d\theta = \lim_{a\to\infty} \int_0^{2\pi}\int_0^{a} \left(-\frac{1}{2}\right)e^{-r^2}\, d(r^2)\, d\theta =$

$\lim_{a\to\infty} \int_0^{2\pi} \left[-\frac{1}{2}e^{-r^2}\right]_{r=0}^{r=a} d\theta = \lim_{a\to\infty} 2\pi\left[-\frac{1}{2}(e^{-a^2}-1)\right]$

$= 2\pi\left(-\frac{1}{2}\right)(-1) = \pi$

(b) $\pi = \iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\, dA = \lim_{a\to\infty} \iint_{S_a} e^{-(x^2+y^2)}\, dA$




$= \lim_{a\to\infty} \int_{-a}^{a}\int_{-a}^{a} e^{-(x^2+y^2)}\, dy\, dx$

$= \lim_{a\to\infty} \int_{-a}^{a} e^{-x^2} \int_{-a}^{a} e^{-y^2}\, dy\, dx$

$= \lim_{a\to\infty} \int_{-a}^{a} e^{-x^2}\, dx \int_{-a}^{a} e^{-y^2}\, dy = \int_{-\infty}^{\infty} e^{-x^2}\, dx \int_{-\infty}^{\infty} e^{-y^2}\, dy$

So: $\int_{-\infty}^{\infty} e^{-x^2}\, dx \int_{-\infty}^{\infty} e^{-y^2}\, dy = \pi$.