MATH 2339   HW33 : Solution

§12.5

2/.

(1) $$\iiint_E (xy+z^2)\,dV = \int_0^2\int_0^1\int_0^3 (xy+z^2)\,dz\,dy\,dx$$

$$= \int_0^2\int_0^1 \left[xyz + \frac{z^3}{3}\right]_{z=0}^{z=3} dy\,dx$$

$$= \int_0^2\int_0^1 (3xy+9)\,dy\,dx$$

$$= \int_0^2 \left[3x\frac{y^2}{2} + 9y\right]_{y=0}^{y=1} dx$$

$$= \int_0^2 \left(3x\frac{1}{2} + 9\right) dx$$

$$= \left[\frac{3}{2}\frac{x^2}{2} + 9x\right]_0^2 = 3 + 18 = 21$$

(2) $$\iiint_E (xy+z^2)\,dV = \int_0^1\int_0^2\int_0^3 (xy+z^2)\,dz\,dx\,dy$$

$$= \int_0^1\int_0^2 (3xy+9)\,dx\,dy$$

$$= \int_0^1 \left[3y\frac{x^2}{2} + 9x\right]_{x=0}^{x=2} dy$$

$$= \int_0^1 (6y+18)\,dy$$

$$= \left[3y^2 + 18y\right]_0^1 = 3 + 18 = 21$$

(3) $$\iiint_E (xy+z^2)\,dV = \int_0^3\int_0^1\int_0^2 (xy+z^2)\,dx\,dy\,dz$$

$$=\int_0^3\int_0^1 \left[y\frac{x^2}{2}+z^2x\right]_{x=0}^{x=2} dy\,dz$$

$$=\int_0^3\int_0^1 (2y+2z^2)\,dy\,dz$$

$$=\int_0^3 \left[y^2+2z^2y\right]_{y=0}^{y=1} dz$$

$$=\int_0^3 (1+2z^2)\,dz$$

$$=\left[z+\frac{2}{3}z^3\right]_0^3 = 3+18 = 21$$

4/. $$\int_0^1\int_0^1\int_0^{\sqrt{1-z^2}} \frac{z}{y+1}\,dx\,dz\,dy = \int_0^1\int_0^1 \left[\frac{z}{y+1}x\right]_{x=0}^{x=\sqrt{1-z^2}} dz\,dy$$

$$=\int_0^1\int_0^1 \frac{z\sqrt{1-z^2}}{y+1}\,dz\,dy = \int_0^1\int_0^1 \frac{\sqrt{1-z^2}}{y+1}\,d(1-z^2)\left(-\frac{1}{2}\right)dy$$

$$=-\frac{1}{2}\int_0^1\left[\frac{1}{y+1}\,\frac{2}{3}(1-z^2)^{\frac{3}{2}}\right]_{z=0}^{z=1} dy$$

$$=-\frac{1}{2}\int_0^1 \frac{1}{y+1}\,\frac{2}{3}(0-1)\,dy = \frac{1}{3}\int_0^1\frac{1}{y+1}\,d(y+1)$$

$$=\frac{1}{3}\Big[\ln|y+1|\Big]_0^1 = \frac{1}{3}(\ln 2-\ln 1) = \frac{1}{3}\ln 2.$$

5/. $$\int_0^{\pi/2}\int_0^{y}\int_0^{x} \cos(x+y+z)\,dz\,dx\,dy$$

$$= \int_0^{\pi/2}\int_0^{y} \Big[\sin(x+y+z)\Big]_{z=0}^{z=x} dx\,dy$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{y} \big[\sin(2x+y) - \sin(x+y)\big]\,dx\,dy$$

$$= \int_0^{\frac{\pi}{2}} \left[-\frac{\cos(2x+y)}{2} + \cos(x+y)\right]_{x=0}^{x=y} dy$$

$$= \int_0^{\frac{\pi}{2}} \left[-\frac{1}{2}(\cos 3y - \cos y) + (\cos 2y - \cos y)\right] dy$$

$$= \int_0^{\frac{\pi}{2}} \left(-\frac{1}{2}\cos 3y + \cos 2y - \frac{1}{2}\cos y\right) dy$$

$$= \left[-\frac{1}{2}\,\frac{\sin 3y}{3} + \frac{\sin 2y}{2} - \frac{1}{2}\sin y\right]_0^{\frac{\pi}{2}}$$

$$= -\frac{1}{2}\,\frac{-1}{3} + 0 - \frac{1}{2} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$$