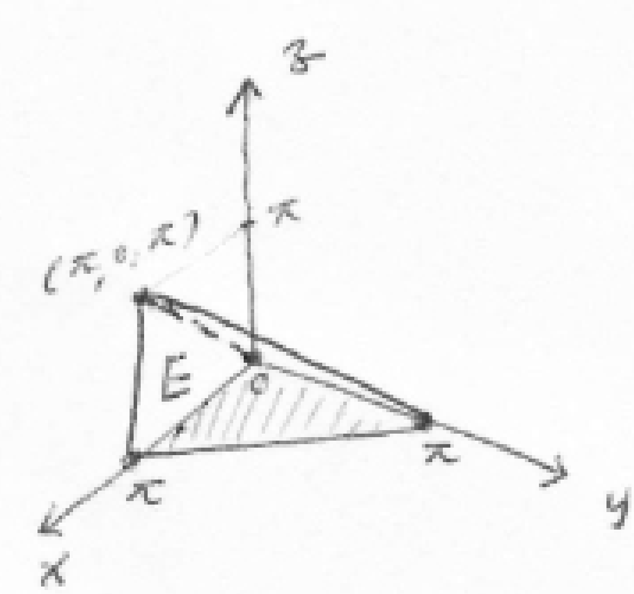
MATH 2339, HW34: Solution

§12.5

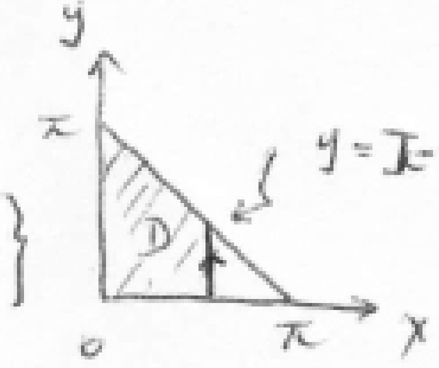
10/. $I = \iiint_E \sin y \, dV$



$$= \iint_D \left(\int_0^x \sin y \, dz \right) dA$$

where: $D = \{ (x,y) \mid 0 \le x \le \pi,\ 0 \le y \le \pi - x \}$

$E = \{ (x,y,z) \mid 0 \le x \le \pi,\ 0 \le y \le \pi - x,\ 0 \le z \le x \}$



So: $$I = \int_0^\pi \int_0^{\pi - x} \int_0^x \sin y \, dz \, dy \, dx$$

$$= \int_0^\pi \int_0^{\pi - x} \Big[(\sin y)\, z \Big]_{z=0}^{z=x} dy \, dx$$

$$= \int_0^\pi \int_0^{\pi - x} (\sin y)(x) \, dy \, dx$$

$$= \int_0^\pi x \Big[-\cos y \Big]_{y=0}^{y=\pi - x} dx$$

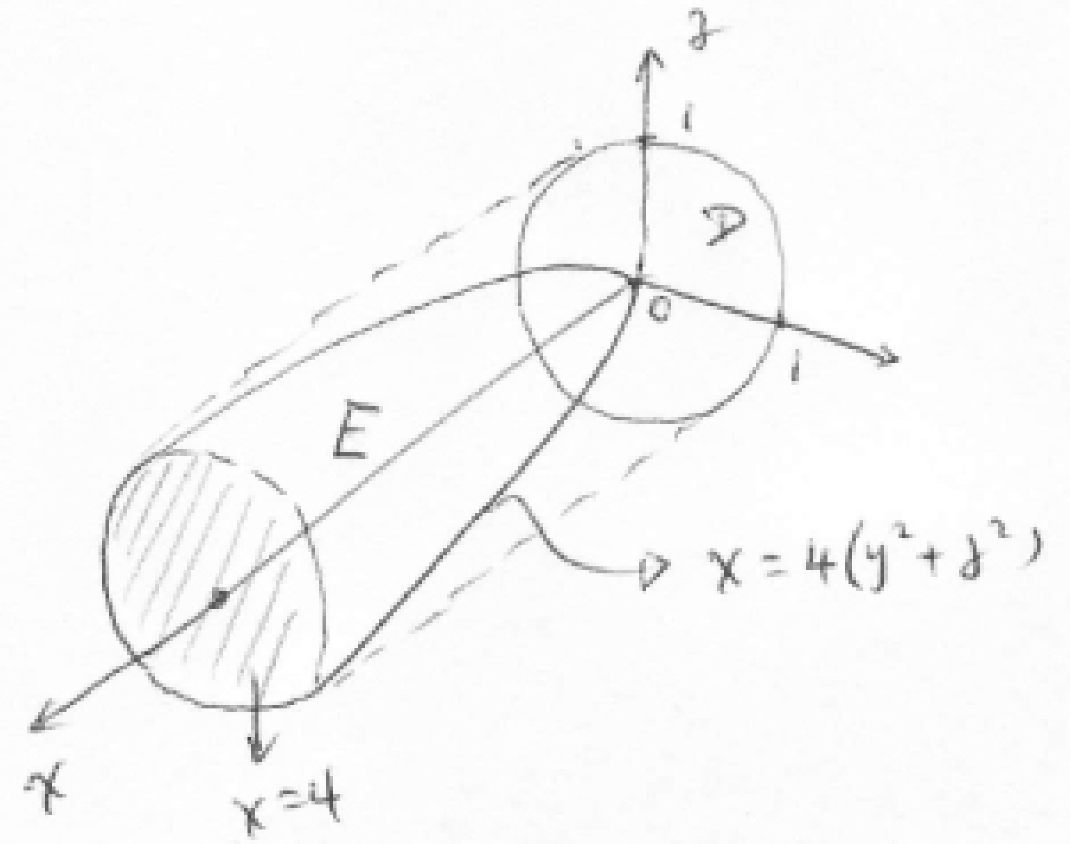
$$= \int_0^\pi -(x) \Big[\underbrace{\cos(\pi - x)}_{= -\cos x} - \underbrace{\cos 0}_{=1} \Big] dx$$

$$= \int_0^\pi x(\cos x + 1) \, dx = \underbrace{\int_0^\pi x \overbrace{\cos x}^{\frac{d}{dx}(\sin x)} dx}_{\text{int. by parts}} + \Big[\tfrac{1}{2} x^2 \Big]_0^\pi$$

$$= \Big[x \sin x \Big]_0^\pi - \int_0^\pi \sin y \, dx + \tfrac{1}{2}\pi^2$$

$$= \tfrac{1}{2}\pi^2 + \Big[\cos x \Big]_0^\pi = \tfrac{1}{2}\pi^2 - 2$$

15/. $$I = \iiint_E x \, dV$$



$$\begin{cases} x = 4 \\ x = 4(y^2+z^2) \end{cases} \Rightarrow \text{intersection: } \begin{cases} x = 4 \\ 4 = 4(y^2+z^2) \end{cases}, \text{ i.e. } y^2+z^2 = 1$$

So the projection of E on the yz plane is:

$$D: \quad y^2 + z^2 \le 1$$

The solid E is: $E = \{ (x, y, z) \mid (y, z) \in D, \ 4(y^2+z^2) \le x \le 4 \}$

$$I = \iiint_E x \, dV = \iint_D \left(\int_{4(y^2+z^2)}^{4} x \, dx \right) dA$$

$$= \iint_D \left[\frac{1}{2} x^2 \right]_{x=4(y^2+z^2)}^{x=4} dA = \frac{1}{2} \iint_D \left[16 - 16(y^2+z^2)^2 \right] dA$$

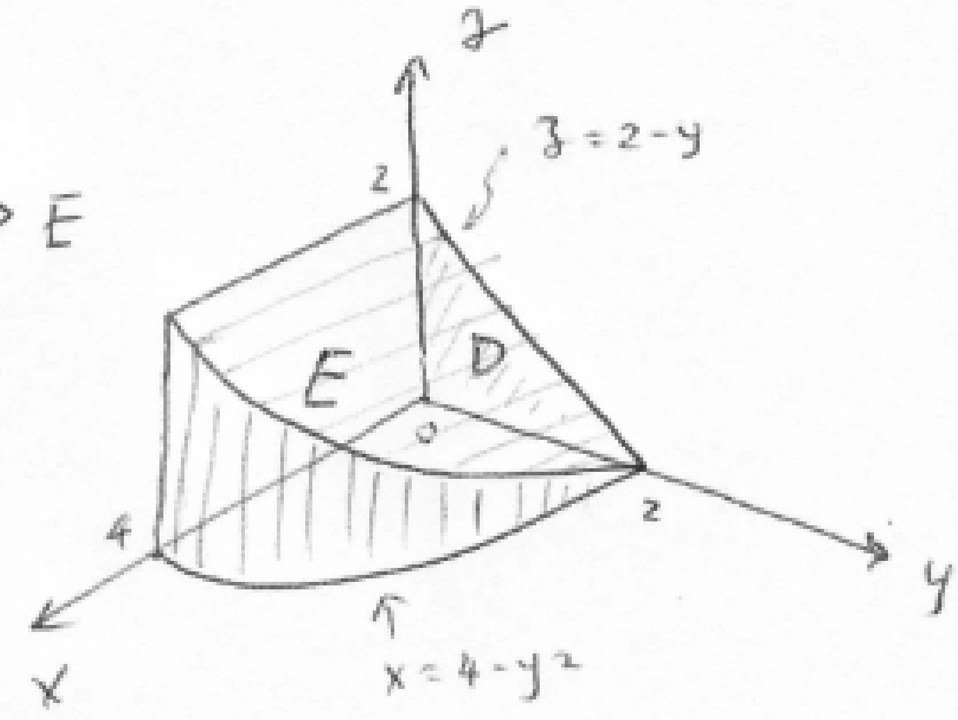
Using polar coordinates in the yz plane for D:

$$I = \frac{16}{2} \int_0^{2\pi} \int_0^1 (1 - r^4) \, r \, dr \, d\theta = 8 \cdot 2\pi \cdot \left[\frac{r^2}{2} - \frac{r^6}{6} \right]_0^1$$

$$= 16\pi \left(\frac{1}{2} - \frac{1}{6} \right) = \frac{16\pi}{3}.$$

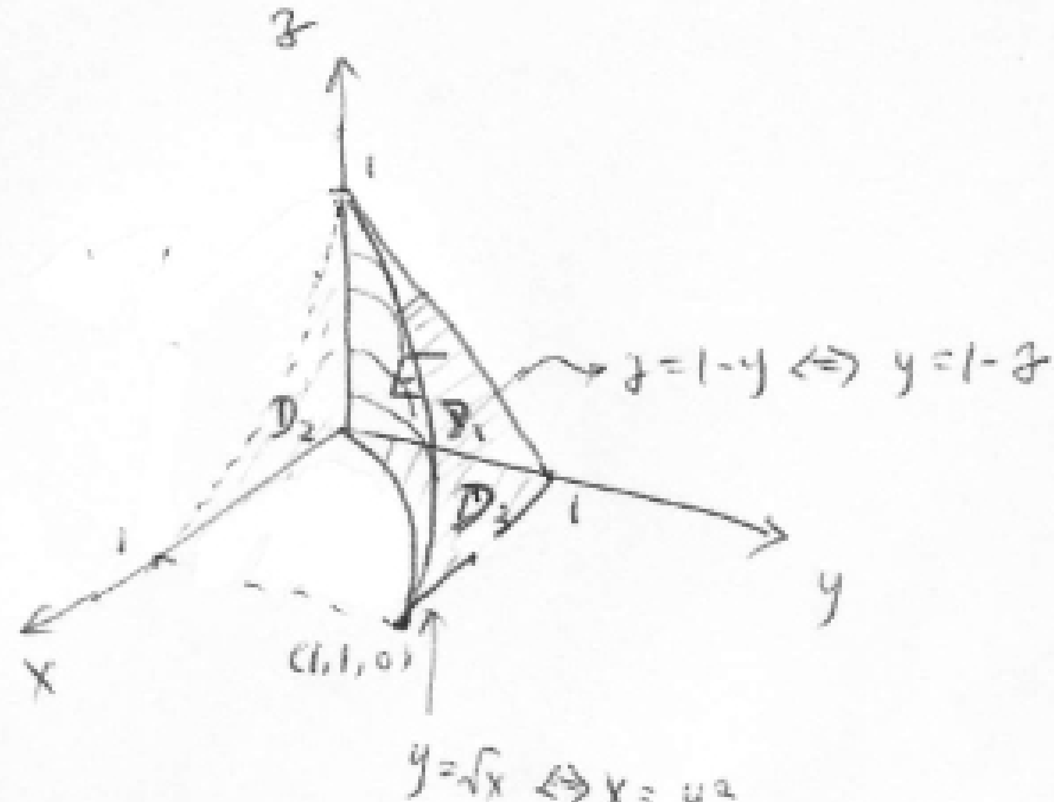
26/. $\displaystyle\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx\, dz\, dy$

$$\left.\begin{array}{l} \left.\begin{array}{l} y: 0 \to 2 \\ z: 0 \to 2-y \end{array}\right\} \Rightarrow D \\ x: 0 \to 4-y^2 \end{array}\right\} \Rightarrow E$$

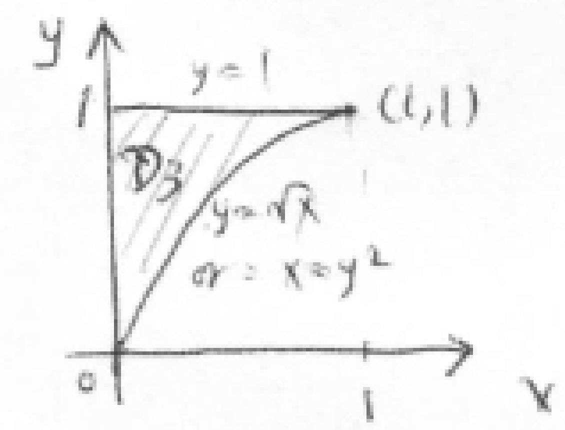


31/. $\displaystyle I = \int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\, dz\, dy\, dx$

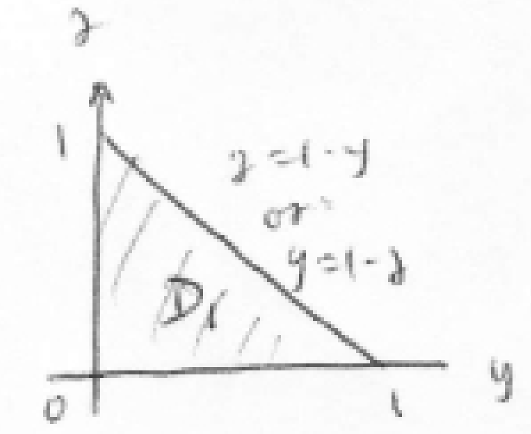
$$\left.\begin{array}{l} \left.\begin{array}{l} x: 0 \to 1 \\ y: \sqrt{x} \to 1 \end{array}\right\} \Rightarrow D_3 \\ z: 0 \to 1-y \end{array}\right\} \Rightarrow E$$

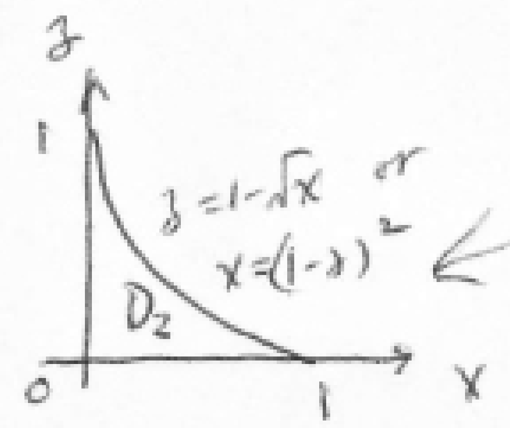


projections of the solid E:



intersection: $\begin{cases} z = 1-y \\ y = \sqrt{x} \end{cases} \Rightarrow$ projection





So: