Homework #4 MATH 309 - Solutions

Sec. 3.1 1

$x_1 = (8,6)^T$, $x_2 = (4,-1)^T$ in $\mathbb{R}^2$

1(a) $|x_1| = \sqrt{8^2+6^2} = \sqrt{100} = \boxed{10}$

$|x_2| = \sqrt{4^2+(-1)^2} = \boxed{\sqrt{17}}$

1(b) $x_3 = (12,5)^T$

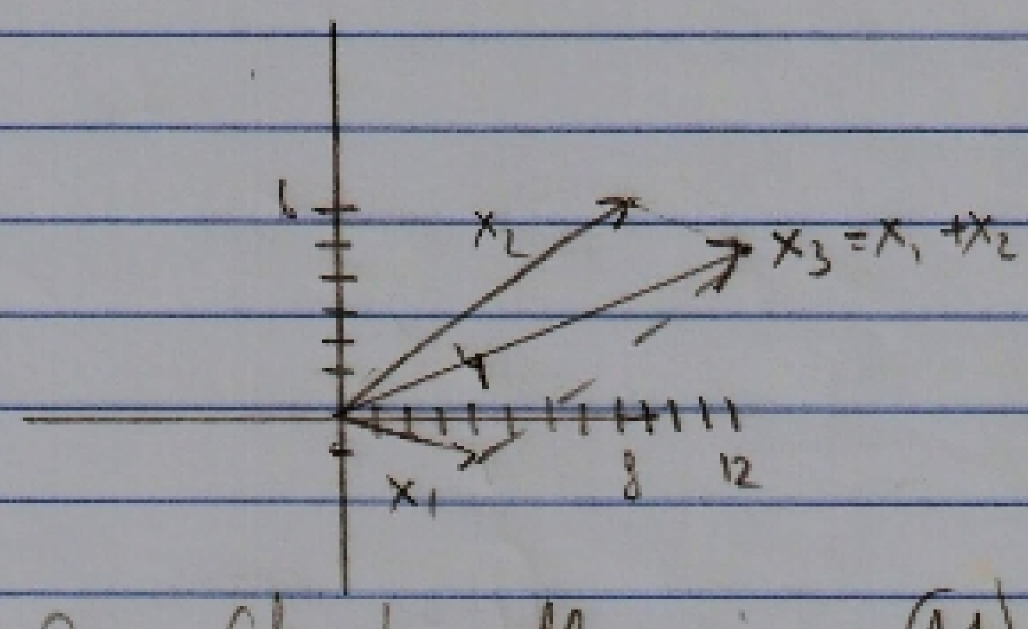
$|x_3| = \sqrt{12^2+5^2} = \sqrt{144+25} = \sqrt{169} = \boxed{13}$

$|x_1|+|x_2| = 10 + \underbrace{\sqrt{17}}_{>4} > 14 > 13 = |x_3| \Rightarrow |x_1|+|x_2| > |x_3|$

(In general, there is the triangle inequality

$$|x_1+x_2| \leq |x_1|+|x_2|)$$

(c)



Sec3.1 3 Problem 3 Check all axioms (A1)-(A8)

(A1) $x = a+bi$, $y = c+id$

$x+y = (a+c)+(b+d)i$ Since $a+c=c+a$ and $b+d=d+b$

$y+x = (c+a)+(d+b)i$ then $x+y = y+x \Rightarrow$ (A1) holds

(here we used commutativity of reals under addition)

(A2) $x = a + bi,\ y = c + di,\ z = e + if$

$(x+y)+z = (a+c)+e + ((b+d)+f)i =$

$= a + (c+e) + (b+(d+f))i = x + (y+z) \Rightarrow$ (A2) hold

(Here we used associativity of reals under addition)

(A3) let $0 = 0 + 0i \Rightarrow$ for any $x = a + ib$

$x + 0 = 0 + x = a + bi = x \Rightarrow$ (A3) hold

(A4) For $x = a + bi$ take $y = (-a) + i(-b) \Rightarrow$

$x + y = y + x = 0 \Rightarrow y$ can be taken as $-x$

(A5) For $x = a + bi$ and $y = c + di$

$\alpha(x+y) = \alpha((a+c) + (b+d)i) = \alpha(a+c) + \alpha(b+d)i$

$= \alpha a + \alpha c + (\alpha b + \alpha d)i = (\alpha a + \alpha bi) + (\alpha c + \alpha di) =$

$= \alpha(a + bi) + \alpha(c + di) = \alpha x + \alpha y$

(here we used the distributive law for reals)

(A6) Let $x = a + bi \Rightarrow (\alpha+\beta)x = (\alpha+\beta)(a + bi) =$

$= (\alpha+\beta)a + (\alpha+\beta)bi = (\alpha a + \beta a) + (\alpha b + \beta b)i =$

$= (\alpha a + \alpha bi) + (\beta a + \beta bi) = \alpha(a + bi) + \beta(a + bi) = \alpha x + \beta x$

(here we used the distributive law for reals)

(A7) Let $x = a+bi$

$$(\alpha\beta)x = (\alpha\beta)(a+bi) = (\alpha\beta)a + (\alpha\beta)bi =$$

$$= \alpha(\beta a) + \alpha(\beta b)i = \alpha(\beta a + \beta bi) = \alpha(\beta(a+bi)) = \alpha(\beta x)$$

(Here we used associativity of the multiplication of reals)

(A8) $1 \cdot x = x$ for all $x \in V$

$$1 \cdot (a+bi) = 1 \cdot a + 1 \cdot bi = a+bi = x$$

Sec 3.1 7. Assume that we have two elements 0_I and 0_{II} such that

$$x + 0_I = 0_I + x = x \quad \text{for any } x \qquad (1)$$

$$x + 0_{II} = 0_{II} + x = x \quad \text{for any } x \qquad (2)$$

In (1) take $x = 0_{II} \Rightarrow$

$$0_{II} + 0_I = 0_I + 0_{II} = 0_{II}$$

On the other hand from (2)

$$0_{II} + 0_I = 0_I \Rightarrow 0_I = 0_{II}, \text{ i.e}$$

zero element is unique.

Sec 3.1 10. Axiom A13) (an existence of zero element) does not hold. Indeed assume that 0 is a zero element (note that a priori we don't know whether $0 = (0,0)$)