

CSCI 570 - Fall 2021 - HW 2

Due September 9th

1 Graded Problems

1. What is the worst-case runtime performance of the procedure below?

```
c = 0
i = n
while i > 1 do
  for j = 1 to i do
    c = c + 1
  end for
  i = floor(i/2)
end while
return c
```

2. Arrange these functions under the O notation using only $=$ (equivalent) or \subset (strict subset of):

- (a) $2^{\log n}$
- (b) 2^{3n}
- (c) $n^{n \log n}$
- (d) $\log n$
- (e) $n \log(n^2)$
- (f) n^{n^2}
- (g) $\log(\log(n^n))$

E.g. for the function n , $n + 1$, n^2 , the answer should be

$$O(n + 1) = O(n) \subset O(n^2).$$

3. Given functions f_1, f_2, g_1, g_2 such that $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$. For each of the following statements, decide whether you think it is true or false and give a proof or counterexample.
- (a) $f_1(n) \cdot f_2(n) = O(g_1(n) \cdot g_2(n))$
 - (b) $f_1(n) + f_2(n) = O(\max(g_1(n), g_2(n)))$
 - (c) $f_1(n)^2 = O(g_1(n)^2)$
 - (d) $\log_2 f_1(n) = O(\log_2 g_1(n))$
4. Given an undirected graph G with n nodes and m edges, design an $O(m+n)$ algorithm to detect whether G contains a cycle. Your algorithm should output a cycle if G contains one.

2 Practice Problems

1. Solve Kleinberg and Tardos, **Chapter 2, Exercise 6**.
2. Solve Kleinberg and Tardos, **Chapter 3, Exercise 6**.