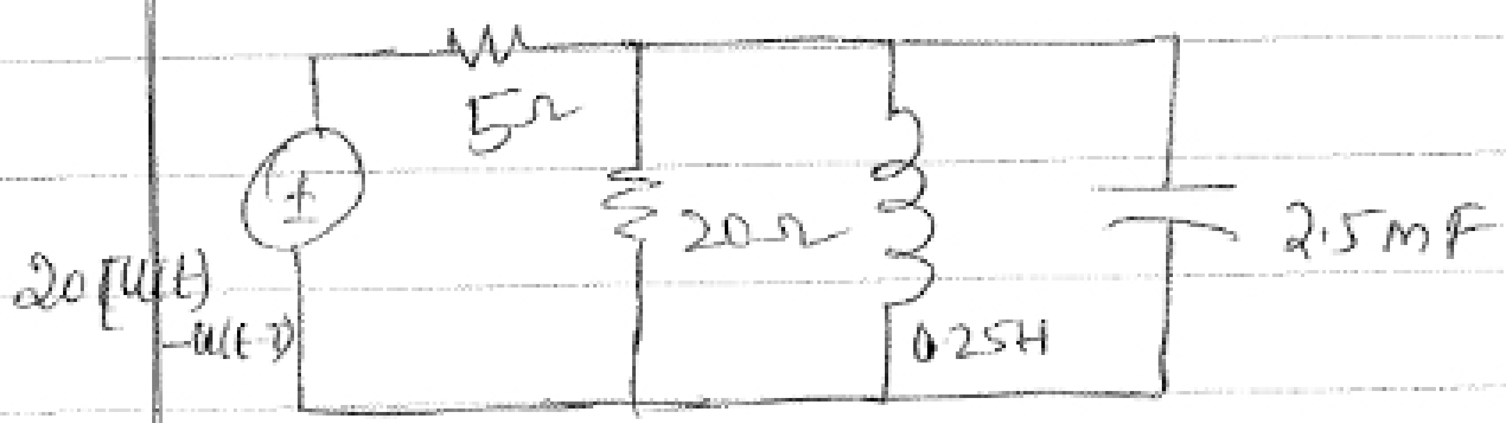
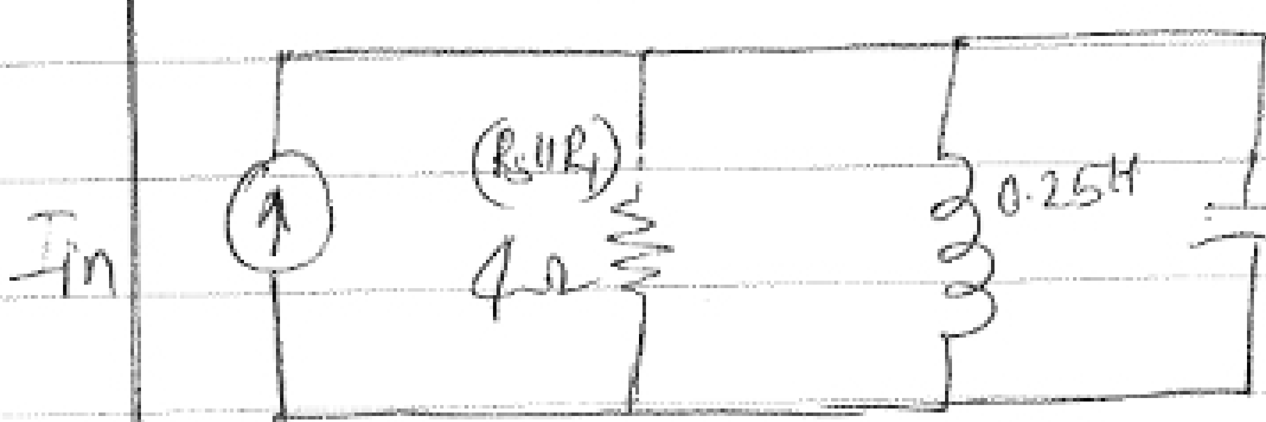
HW – 25

Chap-9 37



⇓

$$I_{in} = \frac{V_{in}}{R_s}$$



$I_{in} = 4[u(t) - u(t-0.25)]$

Characteristic eqn:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$\Rightarrow s^2 + 100s + 1600 = 0$$

roots $s_1, s_2 = -20, -80$: real & distinct

$\Rightarrow$ overdamped response.

General form of soln $x(t) = Ae^{+s_1 t} + Be^{+s_2 t} + X_F$

For $\boxed{t : 0 < t < 0.25}$

Zero state response $i_L(0) = 0$ $V_C(0) = 0$ $I_{in} = 4$

$i_L(t) = Ae^{-20t} + Be^{-80t} + X_F$: $X_F = i_L(\infty) = 4$

$\Rightarrow i_L(t) = Ae^{-20t} + Be^{-80t} + 4$

$i_L(0) = A + B + 4 = 0 \Rightarrow \boxed{A + B = -4}$ —(1)

$V_C(t) = V_L(t) = L\frac{di_L}{dt} = -5Ae^{-20t} - 20e^{-80t} \cdot B$

$V_C(0) = 0 \Rightarrow 0 = -5A - 20B \Rightarrow \boxed{-4B = A}$ —(2)

Solving ① & ② we get $B = \frac{4}{3}$ $A = \frac{-16}{3}$

$\therefore$ Zero state response $0 < t < 0.25$

$$\boxed{\begin{aligned} i_L(t) &= \frac{-16}{3} e^{-20t} + \frac{4}{3} e^{-80t} + 4 \\ V_C(t) &= \frac{80}{3} e^{-20t} - \frac{80}{3} e^{-80t} \end{aligned}} \Rightarrow 0 < t < 0.25$$

Zero i/p response : $0 < t < 0.25$

$I_{in} = 0$ From circuit conditions before $t=0$

$$i_L(0^-) = 0 = i_L(0^+)$$

$$V_C(0^+) = 0 = V_C(0^+)$$

$i_L(t) = Ae^{-20t} + Be^{-80t} + X_F \quad X_F = 0$ (as its Z.I res)

$\Rightarrow i_L(t) = Ae^{-20t} + Be^{-80t}$

$i_L(0) = 0 \Rightarrow A = -B$ —— (3)

$V_C(t) = -5Ae^{-20t} + (-20)Be^{-80t}$

$V_C(0) = 0 \Rightarrow A = -4B$ —— (4)

Eqn (3) & (4) have only trivial soln @ $A=0$ & $B=0$

$\therefore$ Zero i/p response : $0 < t < 0.25$

$$\boxed{\begin{aligned} i_L(t) &= 0 \\ V_C(t) &= 0 \end{aligned}} \Rightarrow 0 < t < 0.25$$

This can be intuitively realized as the initial conditions are zero the zero i/p response has to be zero, as there is nothing in the circuit to have a response.

$\therefore$ Complete response = Z.S + Z.I response

For t: $0 < t < 0.25$

$\Rightarrow$

$$\boxed{\begin{aligned} i_L(t) &= \frac{-16}{3} e^{-20t} + \frac{4}{3} e^{-80t} + 4 \\ V_C(t) &= \frac{80}{3} e^{-20t} - \frac{80}{3} e^{-80t} \end{aligned}}$$

For t : $\boxed{t > 0.25}$ Still the circuit is overdamped as R, L, C values are the same

<u>Zero input response</u> $I_{in} = 0$

We have to find initial cond @ $t = 0.25$

$V_C(0.25) = 0.1796$

$i_L(0.25) = 3.964$

$I_{in} = 0$

$\Rightarrow X_F = I_L(\infty) = 0$

$i_L(t) = Ce^{-20(t-0.25)} + De^{-80(t-0.25)} + X_F$

$i_L(t) = Ce^{-20(t-0.25)} + De^{-80(t-0.25)}$

@ $t = 0.25$ $\quad i_L(0.25) = \boxed{3.964 = C + D}$ —— (5)

$V_C(t) = L\dfrac{di_L}{dt} = -5Ce^{-20(t-0.25)} - 20e^{-80(t-0.25)} \cdot D$

$V_C(0.25) = 0.1796 = -5C - 20D$ —— (6)

Solving (5) & (6) $\quad D = -1.333 \quad C = 5.3$

$\therefore$ Zero i/p response

$t > 0.25 \Leftarrow$ $$\boxed{\begin{aligned} i_L(t) &= 5.3e^{-20(t-0.25)} - 1.333e^{-80(t-0.25)} \\ V_C(t) &= -26.5e^{-20(t-0.25)} + 26.666e^{-80(t-0.25)} \end{aligned}}$$

<u>Zero state response</u> : $i_L(0.25) = 0 \quad V_C(0.25) = 0$

$\Rightarrow i_L(t) = Ce^{-20(t-0.25)} + De^{-80(t-0.25)}$ $\quad X_F = 0$

as $I_{in} = 0$ after 0.25

$\Rightarrow i_L(0) = 0 \Rightarrow C = -D$ —(7)

$V_C(t) = -5Ce^{-20(t-0.25)} - 20De^{-80(t-0.25)}$

$\Rightarrow V_C(0) = 0 \Rightarrow C = -4D$ —(8)

$I_{in} = 4[u(t) - u(t-0.25)]$

after $t > 0.25$

$I_{in} = 0$

Eqn (7) & (8) has just the trivial soln $C = 0 \quad D = 0$

$\Rightarrow \boxed{\begin{aligned} i_L(t) &= 0 \\ V_C(t) &= 0 \end{aligned}}$ Zero state response

$t > 0.25$

This can also be intuitively seen, as with zero i/ps, your zero state response has to be zero and its mathematically shown