# AAE 340
# Homework #4
# Due Friday, Sept. 15, 2017 at 9:30 AM
# (Note: HW turned in after 9:45 AM will be given a zero.)

## 1. Forced Vibration Problem

In Lecture 8 (p. 52), the EOM for the forced motion of the mass-spring-damper system is

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{f_o}{m}\cos\omega t \qquad (90)$$

**1a.** Put Eq. (90) into state variable form.

**1b.** Write a MATLAB program to numerically integrate Eq. (90) for the following conditions and parameters: $x(0) = 0$, $\dot{x}(0) = 0$, $\omega_n = 1$ radian/sec, $(f_o/m) = 1$ unit/sec$^2$, and $\zeta = 1/8$. Attach a listing of your program.

**1c.** Run a simulation of Eq.(90) for the conditions of **1b** for a duration of 10 periods of the natural frequency. Make plots of $(f_o/m)\cos(\omega t)$ vs time and $x(t)$ vs time for the following cases:

i) $\omega = 0$ radians/sec
ii) $\omega = 0.5$ radians/sec
iii) $\omega = 1.0$ radians/sec
iv) $\omega = 1.5$ radians/sec
v) $\omega = 2.0$ radians/sec

Notes: Put time on the horizontal axis. You may put $(f_o/m)\cos(\omega t)$ and $x(t)$ on the same plot if the results can be easily distinguished. Or you may make separate plots of $(f_o/m)\cos(\omega t)$ and $x(t)$ in which case you will have ten plots.

**1d.** Measure the ratios of the steady-state amplitude of $x(t)$ divided by the amplitude of $(f_o/m)\cos(\omega t)$. Show your measurements on each plot. Make a table of your results. From Lecture 8, page 58, we see that these ratios will be AF/ $\omega_n^2$. Since $\omega_n = 1$, these numerical ratios should be consistent with the theoretical values for the amplification factor, AF.

**1e.** Using Eq. (109) in Lecture 8, calculate the AF for the five cases in **1c** and **1d** and put these values in the table you constructed in **1d**.

**1f**. Compute the percent error between your measured ratios and the theoretical values for the AF. Add this error to your table. Comment on whether the error is consistent with what you have done.

**2a.** Find the complete solution (transient plus steady state) for

$$\ddot{x} + \omega_n^2 x = \frac{f_o}{m} \sin \omega_n t \tag{1}$$

Assume that the I.C.s are $x(0) = x_o$ and $\dot{x}(0) = \dot{x}_o$.

**2b.** Use MATLAB to numerically integrate Eq.(1) for $\omega_n = 1$ radian/sec, $(f_o/m) = 1$ meter/sec$^2$, $x(0) = 0$ meter, $\dot{x}(0) = 0$ meter/sec. Make a plot of x(t) vs time for a total time of 6 $\pi$ sec. Make a second plot of x vs t for a total time of 60 $\pi$ sec. Is the system stable or unstable and why? Comment on whether the MATLAB solution supports your assessments.

**2c.** Check your results by making a plot of $x_{numerical}$ minus $x_{analytical}$ versus time for 6 $\pi$ sec. (Note: $x_{analytical}$ is the solution you obtained in **2a**, $x_{numerical}$ is the solution obtained in **2b**.)

**2d.** What is the main source of error obtained in **2c**?

**3.** A cart, P, of mass m moves on rails between two springs that are unstressed when R = 0. The rails are on a circular platform that rotates at constant rate, $\Omega$. A uniform gravity field acts on the system. Assume that the sides of the rails have a coefficient of friction, $\mu$, but that the floor of the platform is smooth. Find the EOM. **No credit for not including friction,**

since the frictionless case is already given in Lecture 12.