AAE 340
Homework #5
Due Friday, Sept. 29, 2017 at 9:30 AM
After a 15-minute grace period late HW will be given a zero.

1. Derive the equations of motion for a satellite in orbit and identify the integrals of motion as follows.

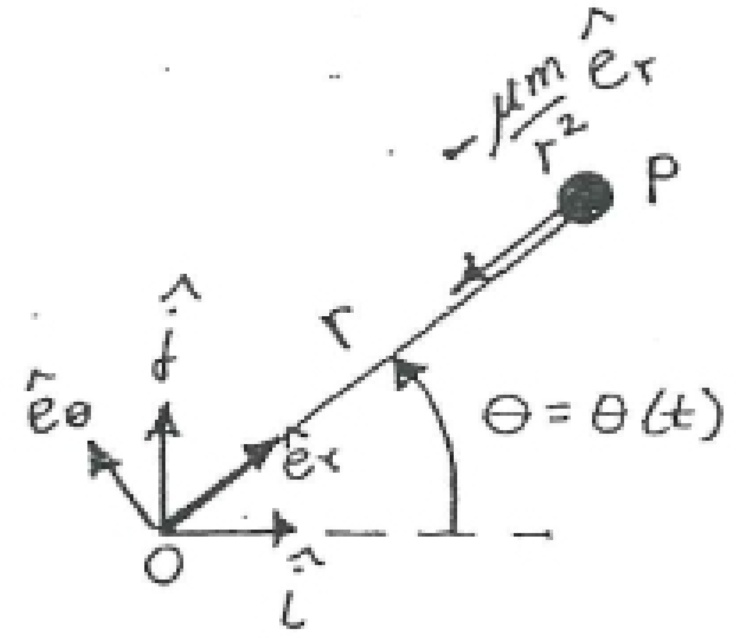
1a. Find the EOMs in four steps, as discussed in the lecture:

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2} \tag{39}$$

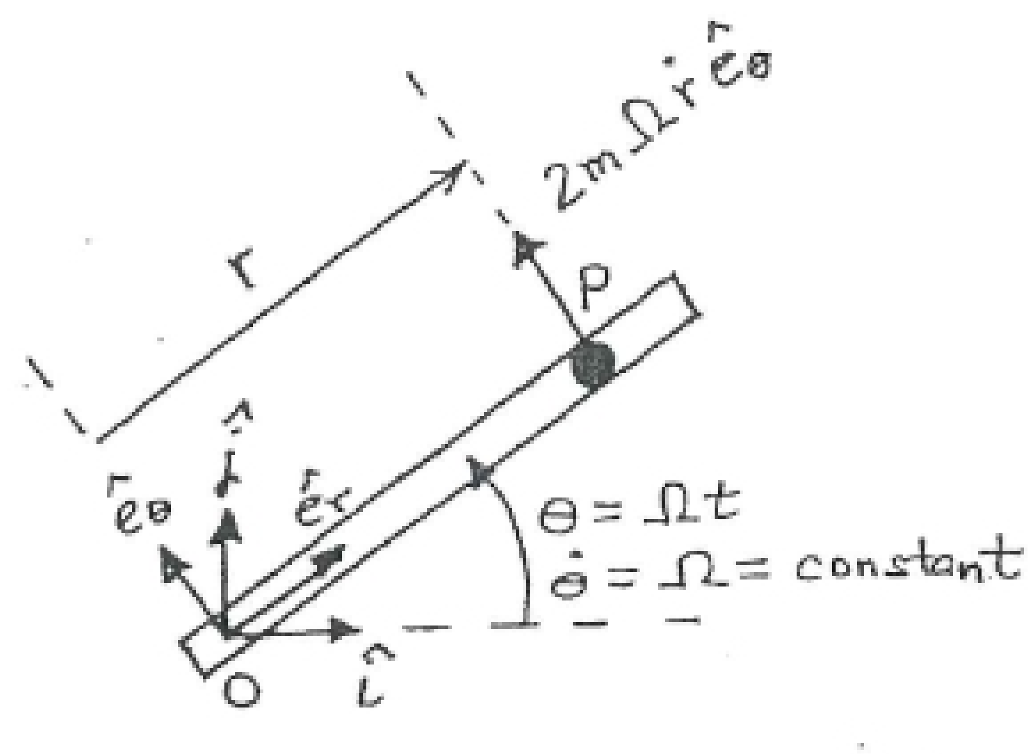
$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \tag{40}$$

1b. Identify the three integrals of the motion. Explain why the satellite motion remains in an inertially-fixed orbital plane. Write an expression for specific angular momentum in terms of the flight path angle, γ. To receive credit: you must provide a logical reason (a proof) that these expressions apply.

2. Work on a Particle



1) Particle, P, in orbit about planet at O.



2) Particle, P, in a spinning frictionless tube.

Consider two problems: 1) a particle, P, in orbit about a massive planet at O and 2) a particle which slides in a frictionless, spinning tube. In both cases we have

$$\vec{R}^{op} = r\hat{e}_r$$

$$^{i}\dot{\vec{R}}^{op} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

The only force acting on P in 1) is gravity, $-(\mu m/r^2)\hat{e}_r$; the only force in 2) is the constraint force, $2m\Omega\dot{r}\hat{e}_\theta$.

2a. Show that in 1) the work done in moving the particle from $r(t_1) = r_1$ to $r(t_2) = r_2$ is $\mu m(r_2^{-1} - r_1^{-1})$.

2b. Show that in 2) the work done in moving the particle from r_1 to r_2 is $m\Omega^2(r_2^2 - r_1^2)$.

3. Perform a simulation in MATLAB of an elliptic transfer orbit from a LEO (Low Earth Orbit) circular orbit at 300 km altitude to a GEO (Geosynchronous Earth Orbit), a circular orbit with a 24-hour period.

3a. Give the EOMs in state variable form.

3b. Derive all initial and final conditions you will need.

3c. Plot the position of the satellite starting from perigee and ending at apogee in Cartesian coordinates. Note: your plot will look better if you make an xy plot where

$$x = r\cos\theta$$
$$y = r\sin\theta$$

and put x on the horizontal axis, y on the vertical axis.

3d. Make a plot which shows the difference between the specific energy computed from ICs and the actual specific energy during the transfer. Comment on the meaning of your plot.

Notes on Problem 3:

i) This is a somewhat open-ended assignment which requires some independent thought. You must calculate your own initial and final conditions. Do not expect your numbers to be exactly the same that other students calculate.

ii) Explain everything you are doing in writing. No credit will be given for unexplained work or numbers.

iii) You may use the following quantities as given:

$R_{Earth} = 6376$ km (radius of Earth)

$\mu = GM = 3.986 \times 10^5\ km^3/s^2$ (gravitational parameter of the Earth)