CSE310 HW05, Thursday, 04/01/2010, Due: Thursday, 04/08/2010

Please note that you have to typeset your assignment using either LaTeX or Microsoft Word. Hand-written assignment will not be graded. You need to submit a hardcopy before the lecture on the due date. You also need to submit an electronic version at the digital drop box. For the electronic version, you should name your file using the format HW05-LastName-FirstName.

1. (10 pts) Suppose you are initially given 8 singleton sets, each containing the numbers $1, 2, \ldots, 8$, respectively. Show the array structure of the sets (you have to show the rank of each root node).

   Solution:

   ```
   i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
   ------------------------------------
   A | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
   ```

   Grading Keys:

   5 pts for correct solution.

   Now we perform `Union`$(1, 2)$, `Union`$(3, 4)$ and `Union`$(2, 4)$, in that order. Show the array structure of the sets (you have to show the rank of each root node) after these operations are performed.

   Solution:

   ```
   i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
   ------------------------------------
   A | 2 | 4 | 4 |-2 | 0 | 0 | 0 | 0 |
   ```

   Grading Keys:

   5 pts for correct solution.

2. (20 pts) Let the disjoint sets have the following structure:

   ```
   i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
   ------------------------------------
   A |-3 | 1 | 1 | 3 | 4 | 3 |-1 | 7 |
   ```

Let's not worry about how we obtained this structure. From now on, `Find-Set` must be performed with path compression and `Link` must be performed by rank.

- If we perform the operation `Find-Set(5)`, what would the structure look like (express it using the array representation).

  Solution:

  ```
  i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
  -----------------------------------
  A |-3 | 1 | 1 | 1 | 1 | 3 |-1 | 7 |
  ```

  Grading Keys:

  4 pts for correct rank of root node;

  4 pts for other correct entries.

- Suppose we have never performed the `Find-Set(5)` operation listed in the above. Instead, we apply the operation `Union(6, 8)`. Show the resulting set structure.

  Solution:

  ```
  i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
  -----------------------------------
  A |-3 | 1 | 1 | 3 | 4 | 1 | 1 | 7 |
  ```

  Grading Keys:

  6 pts for correct path compression;

  6 pts for other correct entries.

3. (10 pts) You are given four matrices A_1, A_2, A_3, A_4 where A_i has P_{i-1} rows and P_i columns, and $P_0 = 50, P_1 = 20, P_2 = 40, P_3 = 30, P_4 = 10$. Use dynamic programming to compute the best way to compute the product $A_1 \times A_2 \times A_3 \times A_4$. You need to compute all entries of the table for the values as well as the table for the splits.

   Solution: Since s[1,4]=1 and s[2,4]=2, we have the optimal solution as follows:

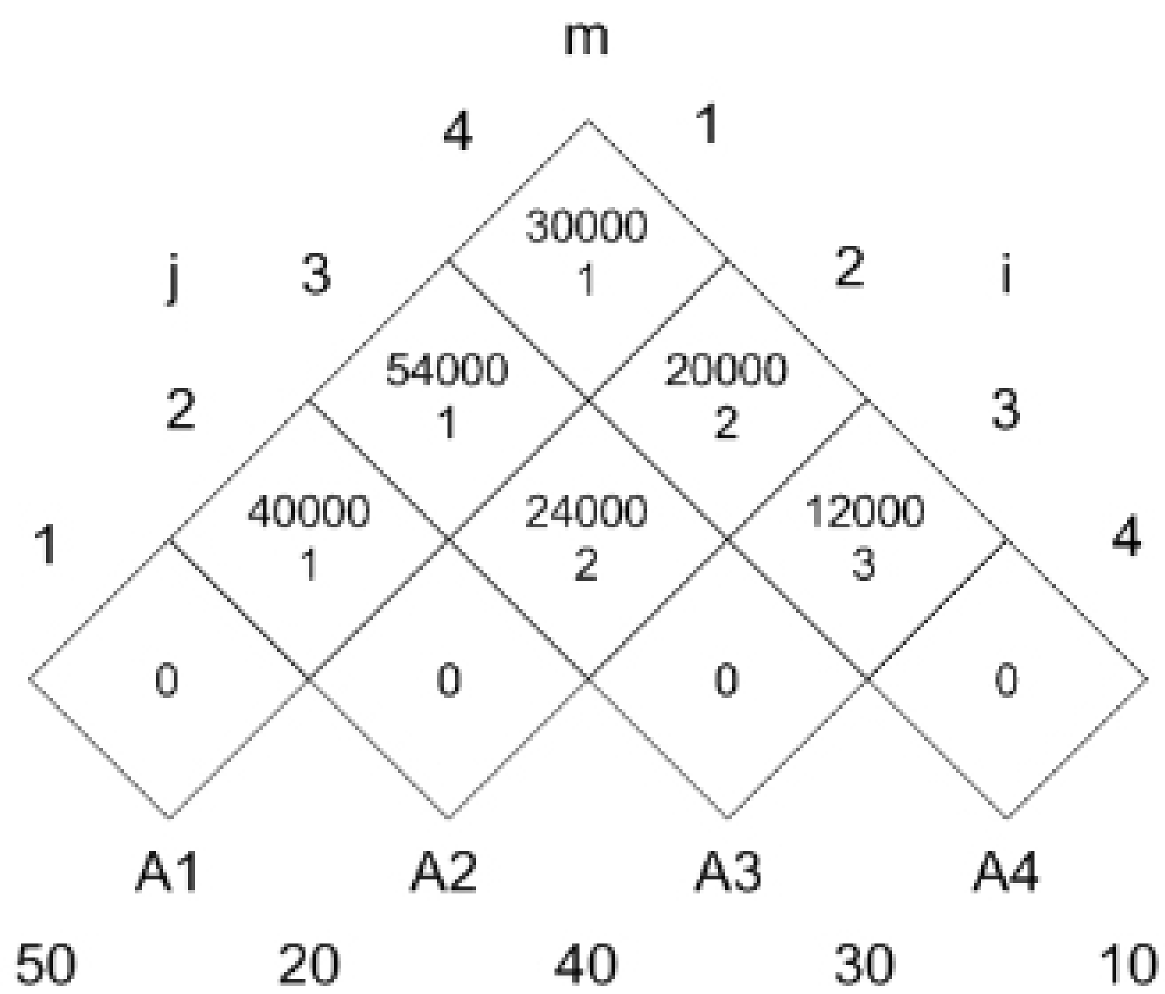
$$A_1 \times (A_2 \times (A_3 \times A_4))$$

   Grading Keys:

   0.5 pt for correct entries of $m[1,2]$, $m[2,3]$ and $m[3,4]$ respectively;

   0.5 pt for correct entries of $s[1,2]$, $s[2,3]$ and $s[3,4]$ respectively;

   1 pts for correct entries of $m[1,3]$ and $m[2,4]$ respectively;



1 pts for correct entries of $s[1,3]$ and $s[2,4]$ respectively;

1.5 pts for correct entry of $m[1,4]$.

1.5 pts for correct entry of $s[1,4]$.

2 pts for correct parenthesization

4. (10 pts) You are given two sequences $X = ABACD$ and $Y = BCA$. Use dynamic programming to compute the LCS of X and Y. You need to show all entries of the table, with X on the left and Y on top.

Solution: The LCS is **BA**

	0 yj=	1 B	2 C	3 A
0 xi=	0	0	0	0
1 A	0	↑0	↑0	↖1
2 B	0	↖1	←1	↑1
3 A	0	↑1	↑1	↖2
4 C	0	↑1	↖2	↑2
5 D	0	↑1	↑2	↑2