

Modern Physics (PHY 3305) Lecture Notes

HomeworkAssignment001

SteveSekula, 21 January 2010 (created 20 January 2010)

Expectations for the quality of your handed-in homework are available at <http://www.physics.smu.edu/sekula/phy3305/homework.pdf>. Failure to meet these guidelines will result in loss of points as detailed in that document. This assignment covers material from Harris Ch. 1, 2.1-2.3 and is worth 100 points.

no tags

MATH AND PHYSICS WARM-UPS

Problem SS-1 (10 Points)

1. Calculate the first derivative with respect to u (that is, dE/du) of the following equation (representing the kinetic energy of a body):

$$E = \frac{1}{2}mu^2$$

2. When a force is applied to an object along the x direction, it is displaced by an amount dx . The *work* (W) done on the object is given by the equation

$$W = \int F dx.$$

The force applied to the object can also be written as the change in momentum with respect to time, $F = dp/dt$, where momentum is defined as the product of the mass and velocity, u , (along the x direction) of the object at any given moment, $p = mu$. The application of the force therefore changes the velocity of the object, where velocity $u = dx/dt$.

- a. Show that the definition of the work done on the object can be rewritten as:

$$W = \int F dx = \int mu du.$$

[HINT: review the "chain rule" of calculus. If a variable x is a function of y , and y is a function of z , you can change variables in a

derivative, such as dx/dy , from y to z by expanding the derivative to $dx/dy = (dx/dz)(dz/dy)$ Also remember that $(dx/dz)dz = dx$]

b. Show that

$$W = \text{constant} + \frac{1}{2}mu^2.$$

[Hint: the result from part 1 of this problem will be helpful]

Problem SS-2 (10 Points)

A physical wave can be described mathematically by the function:

$$\Phi(x, t) = A \cos(2\pi x/\lambda - 2\pi ft),$$

where A is the amplitude of the wave (the maximum height above or below zero that the wave can reach), x and t are locations along the direction of propagation and time since the start of propagation (respectively), and λ and f are the wavelength (distance between two peaks or two troughs in the wave) and the frequency (rate at which two peaks pass the same point in space) of the wave. This function, $\Phi(x, t)$, (the *wave function*) thus tells you the amplitude of the wave at any point in space and at any time.

1. Draw the wave function from $x = 0$ to $x = 2\lambda$, represented as Φ vs. x (this means drawing two axes on your plot, where the vertical axis represents the value of Φ at a given point along the horizontal axis, and the horizontal axis represents a location in x). Label the amplitude, A , and the wavelength, λ , on your drawing of the function.
2. Waves are solutions to an equation known as the *wave equation*. This equation is written as:

$$\frac{1}{u^2} \frac{d^2\Phi}{dt^2} = \frac{d^2\Phi}{dx^2}.$$

Using that equation, and the wave function, solve for the velocity, u , at which the wave propagates.

Harris Conceptual Questions

These refer to "Conceptual Questions" from a chapter or chapters in Harris.

- [CH2-2](#) (5 Points)
- [CH2-3](#) (5 Points)
- [CH2-4](#) (5 Points) [*HINT: keep in mind the definition of proper time when thinking about this question*]

Harris Exercises

These refer to "Exercises" from a chapter or chapters in Harris.

- [CH2-20](#) (10 Points)
- [CH2-21](#) (10 Points)
- [CH2-22](#) (15 Points)
- [CH2-24](#) (15 Points)

Problem **SS-3** (15 Points)

The 24 satellites that orbit the earth and define the Global Positioning System use precision atomic clocks that are accurate to 1×10^{-9} s, or 1 nanosecond (1ns). A person on the earth, using their mobile phone, can usually "see" 4-12 of the 24 satellites at any one time. The satellites are moving, relative to the surface of the earth, at 1.4×10^4 km/h.

1. Calculate the γ_v for one of these satellites. It will be helpful to use the *Binomial Expansion* of the gamma factor,

$$\gamma_v = 1/\sqrt{1 - \frac{v^2}{c^2}} = 1 + \frac{1}{2} \frac{v^2}{c^2} + \frac{1}{8} \frac{v^4}{c^4} + \dots$$

since the numbers involved in the calculation will be quite a lot smaller than 1. Feel free to write your answer either as a sum of terms in the binomial expansion or as $\gamma_v - 1$.

2. Civilian GPS measurements have to be accurate at the level of 5-10m. How accurate must our knowledge of the clocks on the satellites be in order to achieve this level of accuracy in our position measurement? [*HINT: light signals are used by your phone and the satellites to perform the triangulation*]
3. If we measure the passage of 1 day on earth using a clock that is identical to the clocks on the GPS satellites, how much time have the clocks on the satellites measured? What about for the passage of 2 days on earth? For this problem, it will be useful to apply the Binomial