Chapter 5 Homework (Continuous)

14. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} k(x+y) & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

a) Determine $f_X(x)$

b) Determine $f_Y(y)$

c) Determine $f_{X|y}(x)$

d) Determine $f_{X|y=5}(x)$

e) Determine $f_{Y|x}(y)$

f) Determine $f_{Y|x=25}(y)$

g) Determine $E[X]$ and $Var[X]$

h) Determine $E[Y]$

15. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} kxy & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

16. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} k(xy^2) & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

17. Given that $f_{X,Y}(x,y)=\begin{cases} 6x & 0<x<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf. This problem is part Yellow and part Green.

a) Determine $f_X(x)$

b) Determine $f_Y(y)$

c) Determine $f_{X|y}(x)$

d) Determine $f_{X|y=5}(x)$

e) Determine $f_{Y|x}(y)$

f) Determine $f_{Y|x=25}(y)$

g) Determine $E[X]$ and $Var[X]$

h) Determine $E[Y]$

i) Determine $Var[Y]$

j) Determine $E[XY]$

18. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} k(x+y) & 0<x<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

19. Determine k, so that $f_{X,Y}(x,y)=\begin{cases} kxy & 0<x<y<1 \\ 0 & otherwise \end{cases}$ is a joint pdf.

a) Determine $f_X(x)$

b) Determine $f_Y(y)$

c) Determine $f_{X|y}(x)$

d) Determine $f_{X|y=5}(x)$

e) Determine $f_{Y|x}(y)$

f) Determine $f_{Y|x=25}(y)$

g) Determine $E[X]$ and $Var[X]$ h) Determine $E[Y]$ i) Determine $Var[Y]$

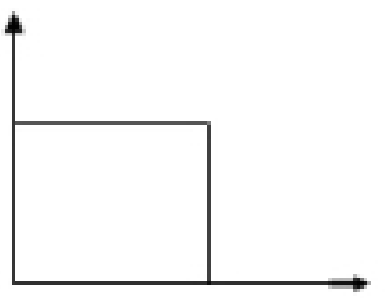
j) Determine $E[XY]$

20. The joint pdf of X and Y is $f_{X,Y}(x,y) = \begin{cases} x+y & 0<x<1,\ 0<y<1 \\ 0 & otherwise \end{cases}$ (region below)

a) Determine $f_X(x)$ b) Determine $f_Y(y)$

c) Determine $f_{X|y}(x)$ d) Determine $f_{X|y=5}(x)$

e) Determine $f_{Y|x}(y)$ f) Determine $f_{Y|x=25}(y)$



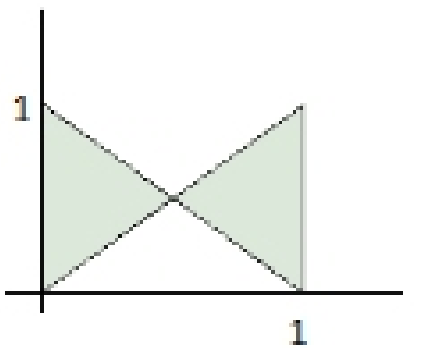
g) Determine $E[X]$ and $Var[X]$ h) Determine $E[Y]$

i) Determine $Var[Y]$ j) Determine $E[XY]$

21. The joint density of the random variables X and Y is $f_{X,Y}(x,y) = 2$ on S, where S is the region bounded by $x=0, x=1, y=x, y=1-x$.

a) Determine $f_Y(y)$ b) Determine $f_X(x)$

c) Determine $E[X]$ and $Var[X]$ d) Determine $E[Y]$ and $Var[Y]$



22. Let X and Y be continuous random variables with joint CDF (not the pdf)

$$F(x,y) = \frac{1}{250}(20xy - x^2y - xy^2) \text{ for } 0 \le x \le 5 \text{ and } 0 \le y \le 5$$

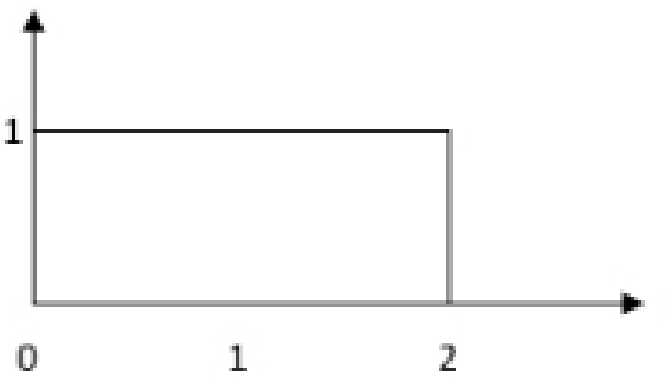
a) Determine $P(3 \le X)$ b) Determine $P(2 \le X)$ c) Determine $P[(2 \le X \le 3) \cap (1 \le Y \le 4)]$

d) Determine $f_X(x)$ e) Determine $f_Y(y)$

23. The joint pdf of X and Y is $f_{X,Y}(x,y) = \begin{cases} \frac{x+y}{3} & 0<x<2,\ 0<y<1 \\ 0 & otherwise \end{cases}$ (region below).

a) Determine $f_{Y|x}(y)$

b) Determine $f_{X|y}(x)$

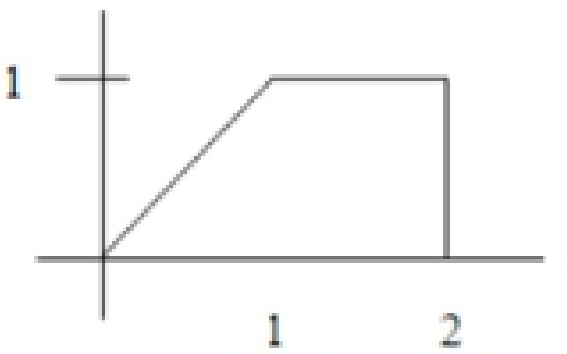


c) Determine $E[Y \mid x]$

d) Determine $E[X \mid y]$

24. Suppose that the joint probability density function of the jointly continuous random variables X and Y is

$$f_{X,Y}(x,y) = \begin{cases} \frac{6}{11}x & \text{on the given region} \\ 0 & \text{otherwise} \end{cases}$$



Determine $f_Y(y)$

Variance and Covariance Problems

The following two formulas will be needed and useful:

1. The Covariance Shortcut Formula: $Cov[X,Y] = E[XY] - E[X] \cdot E[Y]$

2. The Variance of the Sum Formula: $Var[aX + bY] = a^2 Var[X] + b^2 Var[Y] + 2ab\,Cov[X,Y]$

25. Given X and Y have joint density $f_{X,Y}(x,y) = \begin{cases} 6x & 0 < x < y < 1 \\ 0 & otherwise \end{cases}$:

Draw your Support and Random Rectangle

a) Determine $Cov[X,Y]$

$Cov[X,Y] =$ [redacted]

[redacted] = [redacted]

[redacted] = [redacted] = [redacted]