ChE 541 Solution to HW1 M. Sahimi

1. The flux of O_2 is given by

$$N_{O_2} = -D\frac{dC_{O_2}}{dz} + x_{O_2}(N_{O_2} + N_{MO_n})$$

since only diffusion is important, the 2nd term, representing bulk flow or diffusion can be ignored. Thus,

$$N_{O_2} = -D\frac{dC_{O_2}}{dz}$$

The continuity equation for O_2 is given by

$$\frac{\partial C_{O_2}}{\partial t} + \underset{\sim}{\nabla}\cdot N_{O_2} = 0$$

But $\frac{\partial C_{O_2}}{\partial t} \simeq 0$ due to the assumption of quasi-steady-state.

Thus, $\underset{\sim}{\nabla}\cdot N_{O_2} = 0 \Rightarrow \frac{d}{dz}(N_{O_2}) = 0$ substitute for $N_{O_2} \Rightarrow$

$\frac{d}{dz}\left(-D\frac{dC_{O_2}}{dz}\right) = 0 \Rightarrow$ integrate $\Rightarrow C_{O_2} = az + b$.

The boundary conditions are

$$\begin{cases} C_{O_2} = C_0 & z = 0 \\ C_{O_2} = 0 & z = z_f(t) \end{cases} \Rightarrow \text{Thus, } C_{O_2} = -\frac{C_0}{z_f}z + C_0$$

Then, $N_{O_2} = -D \frac{dc_{O_2}}{dz} = D \frac{c_0}{z_f}$

Now, write a mass balance for a region between

$z_f(t)$ and $z_f(t+\Delta t)$ to account for the increase in O_2

$$c_f \cdot x \left[z_f(t+\Delta t) - z_f(t) \right] = N_{O_2} \Delta t$$

substitute for N_{O_2}, divide both side by Δt and let $\Delta t \rightarrow 0$. Then

$$\frac{dz_f}{dt} = \frac{1}{x c_f} \left(D \frac{c_0}{z_f} \right) \Rightarrow z_f \, dz_f = \frac{D c_0}{x c_f} dt$$

given that $z_f = 0$ @ $t = 0$, after integration we obtain

$$\frac{1}{2} z_f^2 = \frac{c_0}{x c_f} D t \longrightarrow z_f(t) = \left(\frac{2 c_0}{x c_f} D t \right)^{1/2}$$

2. (a) The continuity equation for A is given by

$$\frac{\partial C_A}{\partial t} = D\frac{\partial^2 C_A}{\partial z^2} + k C_A^2$$

(b) If we substitute the suggested forms and determine the derivatives, we obtain

$$\frac{\partial C_A}{\partial t} = k\,\frac{\partial C_A}{\partial \tau}$$

and $$\frac{\partial^2 C_A}{\partial z^2} = \left(\frac{k}{D}\right)^2 \frac{\partial^2 C_A}{\partial x^2}$$ $\Rightarrow$ Therefore,

$$\frac{\partial C_A}{\partial \tau} = \frac{\partial^2 C_A}{\partial x^2} + C_A^2$$

(c) Consider $x \longrightarrow x+\beta \Rightarrow \partial x \longrightarrow \partial(x+\beta) = \partial x + \partial\beta$

But $\partial\beta = 0$ (β is a constant) $\Rightarrow$ Thus ∂x doesnt change. similarly for τ: $\partial\tau = \partial(\tau+\alpha) = \partial\tau + \partial\alpha = \partial\tau$

similarly for the second transformation.

$$x \rightarrow \lambda x \rightarrow \partial x \longrightarrow \lambda\partial x,\ \partial x^2 \rightarrow \lambda^2 \partial x^2$$

and $\tau \rightarrow \lambda^2\tau \rightarrow \partial\tau \rightarrow \lambda^2\partial\tau \Rightarrow$ substitute in the equation. We see that we obtain the same equation.