

Econ 154b  
Yale University  
Spring 2005  
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### HOMEWORK #10

*You do not need to turn in this homework assignment but you are responsible for the material it covers. The main purpose of this assignment is to help you study for the final exam.*

1. This problem studies a version of the Solow growth model developed in Chapter 6 of the textbook. Suppose that aggregate output in period  $t$  is given by:  $Y_t = K_t^{0.3}N^{0.7}$ . Labor supply  $N$  is assumed to be constant over time, so without loss of generality set  $N = 1$  throughout the rest of this problem (this is just a normalization of the total labor input and does not affect any of the results). Let the depreciation rate  $d = 0.1$  and let the savings rate  $s = 0.2$ . Aggregate capital evolves according to the following law of motion:

$$K_{t+1} = (1 - d)K_t + sY_t.$$

- (a) Calculate the steady-state capital stock implied by the law of motion for aggregate capital. Calculate the level of steady-state consumption implied by the steady-state capital stock. (Note: The Solow growth model as developed in Chapter 6 of the textbook assumes that the labor force is growing at rate  $n$  and focusses on the dynamic behavior of capital per worker. In this problem, we are assuming that the labor force does not grow: i.e.,  $n = 0$ . Because  $N$  is constant and has been normalized to be equal to 1, total capital,  $K_t$ , and total capital per worker,  $K_t/N$ , are identical. Similarly, total consumption,  $C_t$ , and total consumption per worker,  $C_t/N$ , are identical.)
- (b) Suppose that  $s$  increases to 0.25. What is the new steady-state capital stock? What is the new level of steady-state consumption?
- (c) Describe in qualitative terms the dynamic behavior of the capital stock after the increase in the savings rate (assume that the economy is in a steady state before the increase). Support your answer using an appropriate diagram.
- (d) Show that the golden-rule savings rate (i.e., the value of the savings rate that maximizes steady-state consumption) is equal to the exponent on capital in the aggregate production function (i.e., 0.3). To do this, first express the steady-state capital stock as a function of the savings rate  $s$ . Next, use this result to express the level of steady-state consumption as a function of  $s$ . Finally, take the derivative

of steady-state consumption with respect to  $s$ , set the derivative equal to 0, and solve for the optimal value of  $s$ .

- (c) Suppose again that  $s = 0.2$  and that the economy is in a steady state. Explain what happens to the capital stock if the depreciation rate suddenly increases to 0.15.

2. This problem investigates the relationship between government budget deficits and trade deficits. (Although this problem is related to material in Chapter 5 of the textbook, Chapter 5 is not required reading and you should be able to work this problem without reading it.) Consider a world with two countries. These countries are open in the sense that they are allowed to trade with each other (i.e., net exports in each country could be positive or negative). Let the subscript  $i$  denote country  $i$ , where  $i$  is equal to either 1 or 2. Country  $i$  is described by the following equations:

$$\begin{aligned}\bar{Y}_i &= C_i + I_i + G_i + NX_i \\ C_i &= 0.8(\bar{Y}_i - T_i) \\ I_i &= 210 - 1000r\end{aligned}$$

In each country, full-employment output (as determined by each country's labor market) is fixed at 1000:  $\bar{Y}_1 = \bar{Y}_2 = 1000$ . Suppose too that  $G_1 = G_2 = 200$  and that each country's government has a balanced budget:  $T_i = G_i$  for  $i = 1, 2$ . Notice that the real interest rate  $r$  does not have a subscript: this is because the world is assumed to have a single goods market. The world interest rate  $r$  adjusts so that *world* savings (i.e., the sum of national savings in each of the two countries) and *world* investment (i.e., the sum of investment in each of the two countries) are equilibrated. Because there are only two countries in the world, net exports in each country must sum to 0, i.e.,  $NX_1 + NX_2 = 0$ .

- (a) Find the equilibrium interest rate. Verify that net exports equal 0 in both countries.
- (b) Suppose now that  $G_1$  increases to 250 while  $T_1$  remains fixed. (Assume that Ricardian equivalence does not hold.) Find the new equilibrium interest rate as well as net exports in each country. Verify for each country that  $S_i = I_i + NX_i$ , where  $S_i \equiv \bar{Y}_i - C_i - G_i$  is national saving in country  $i$ . How is the government budget deficit in country 1 financed? (Recall from Chapter 2 that a country with a negative current account balance is, in effect, borrowing from the rest of the world. Note too that we are implicitly assuming in this problem that net foreign payments for each country are equal to 0. Thus, in this problem, the current account is equal to net exports; a country with negative net exports has a trade deficit and hence a negative current account.)

- (c) Now suppose that government spending in *both* countries rises to 250 (again, taxes do not change and Ricardian equivalence does not hold in either country). Find the new equilibrium interest rate. How are the government budget deficits in the two countries financed in this case?
- (d) Suppose now that the two countries are closed: they cannot trade with each other (net exports are forced to equal zero in both countries) and there is a separate goods market in each country, with possibly different equilibrium interest rates. What happens when  $G_1$  increases from 200 to 250? How is the government budget deficit in country 1 financed in this case? How do your answers compare to your answers in parts (b) and (c)?