Econ 510a (second half)
Yale University
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Prof. Tony Smith

HOMEWORK #6

You do not need to turn in answers to this homework but you are responsible for the material it covers.

1. Introduce a "pay-as-you-go" social security system into the Diamond overlapping generations model that we developed in lecture on Wednesday, December 1. In particular, suppose that in every period the government taxes the labor income of the young at rate τ and gives the proceeds to the old.

(a) Assume time-separable utility with logarithmic felicity function and discount factor β, Cobb-Douglas production with capital's share α, and full depreciation of the capital stock in one period. Solve explicitly for the equilibrium law of motion of the aggregate capital stock.

(b) Show that the introduction of social security leads to a downward shift in the equilibrium law of motion: for any (positive) level of capital today, capital tomorrow is lower in a world with social security than in a world without.

(c) Suppose $\tau = 0$, $\beta = 3/4$, and $\alpha = 1/4$. Is the steady state dynamically efficient?

(d) For what value of τ is the steady-state capital stock equal to its "golden-rule" value?

2. Introduce a government that borrows and lends into the neoclassical growth model (set the growth rate equal to zero). Specifically, the government seeks to finance a fixed (deterministic) stream of expenditures $\{g_t\}_{t=0}^{\infty}$; these expenditures are not valued by consumers. The government issues a sequence of one-period debt $\{b_t\}_{t=0}^{\infty}$, with $b_0 = 0$. This debt is a promise by the government to pay one unit of the consumption good in the next period; let the price of such a promise in period t be q_t. The government also engages in lump-sum taxation of consumers; let τ_t be the lump-sum tax in period t. In every period, the government satisfies the following budget constraint:

$$\tau_t + q_t b_{t+1} = b_t + g_t.$$

The left-hand side of this constraint is the government's inflows in period t (measured in terms of period-t consumption goods), while the right-hand side is the government's outflows in period t (again, measured in terms of period-t consumption goods).

The representative consumer takes prices as given and seeks to maximize the lifetime utility of consumption subject to a lifetime budget constraint given by:

$$\sum_{t=0}^{\infty} p_t(c_t + k_{t+1} + q_t a_{t+1}) = \sum_{t=0}^{\infty} p_t((r_t + 1 - \delta)k_t + w_t + a_t - \tau_t),$$

where p_t is the price of period-t consumption goods in terms of period-0 consumption goods (whose price p_0 is normalized to 1). In this budget constraint, a_{t+1} is the amount of government debt (i.e., promises by the government to deliver consumption goods in period $t+1$) that the consumer purchases in period t (assume that $a_0 = 0$). In equilibrium, the demand for government debt is equal to the supply of government debt in every period: $a_t = b_t$ for all t.

(a) Use the no-arbitrage condition $p_t q_t = p_{t+1}$ to derive the government's consolidated budget constraint:

$$\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} p_t \tau_t.$$

(b) Use the result from part (a) to show that the consumer's lifetime budget constraint can be written as follows:

$$\sum_{t=0}^{\infty} p_t(c_t + k_{t+1} + g_t) = \sum_{t=0}^{\infty} p_t((r_t + 1 - \delta)k_t + w_t).$$

Because the sequences $\{\tau_t\}_{t=0}^{\infty}$ and $\{b_t\}_{t=0}^{\infty}$ do not appear in this budget constraint, it is evident that the way in which the government finances its expenditure stream $\{g_t\}_{t=0}^{\infty}$ is irrelevant to the consumer's optimization problem. Instead, all that matters to the consumer is the net present value of government expenditures, i.e., $\sum_{t=0}^{\infty} p_t g_t$. This implies in turn that the government's financing decisions do not affect the determination of equilibrium prices and quantities. This result is known as the *Ricardian equivalence theorem*.