Homework 30

ECE201: Linear Circuit Analysis

Due in class: Friday, November 11, 2011

Question 1

Add the complex numbers $z_1 = 10e^{-j\pi/4}$ and $z_2 = -5e^{j5\pi/8}$. Express your answer (a) in real and imaginary parts, and (b) as a magnitude and phase. Draw z_1, z_2 and $z_1 + z_2$ on the complex plane. Label the real part, the imaginary part, and magnitude and the phase of $z_1 + z_2$.

Hints (you should be able to solve the previous problem without the hints. Please refrain from reading the hints unless you have invested significant effort).

You are not required to write up answers for the questions in this hint.

1. The basic formula for complex number representation for real values is the Euler's formula: $e^{j\theta} = \cos\theta + j\sin\theta$. This formula allows you to transfer a complex number in polar form into or real and imaginary parts. You need to memorize it for the duration of ECE 201.
2. θ is typically expressed in radians. Notice that π in radians equals to 180°.
3. A complex number corresponds to a point in a "complex plane" where the x-axis is for the real part of the complex number and the y-axis is for the imaginary part.
4. The addition of two complex numbers follow the similar rule as you might have learned in vector algebra.

Question 2

Multiply the complex numbers z_1 = 12-16 j and z_2 = −1+0.75j. Express your answer (a) in real and imaginary parts, and (b) as a magnitude and phase. (c) Compare $|z_1|\,|z_2|$ to $|z_1 z_2|$.

(d) What is the relationship between the phase of z_1, the phase of z_2, and the phase of z_1z_2?

Hints (you should be able to solve the previous problem without the hints. Please refrain from reading the hints unless you have invested significant effort).

You are not required to write up answers for the questions in this hint.

1. The best way to convert a complex number in real and imaginary part to polar form is to place the number in the complex plane. In this way you will visually find out the phase, θ, with no ambiguity.

2. You can then use $\theta = \tan^{-1}(b/a)$ to find the value of , given that you know what quadrant the complex number is in.

3. The absolution value of a complex number, for example $|z_1|$, is the distance from the origin to the point corresponding to the complex number, *i.e.* z_1, in the complex plane. If $z = a + jb$, then $|z_1| = \sqrt{a^2 + b^2}$.

Question 3

For z_1 and z_2 given in Question 2, determine z_1/z_2. Express your answer (a) in real and imaginary parts, and (b) as a magnitude and phase. (c) Compare $|z_1|$ and $|z_2|$ to $|z_1/z_2|$. (d) What is the relationship between the phase of z_1, the phase of z_2, and the phase of z_1/z_2?

Hints (you should be able to solve the previous problem without the hints. Please refrain from reading the hints unless you have invested significant effort).

You are not required to write up answers for the questions in this hint.

1. This question introduces a very important concept, *conjugate* of a complex number. The definition is very simple: if $z = a + jb$, then its conjugate, $z^* = a - jb$. You need to memorize this definition and be able to apply it in ECE 201.
2. Immediately following hint #3 in Question 2, we found that the absolution value of a complex number and its conjugate are equal: $|z| = |z^*|$. Can you prove it?
3. What is the product of a complex number and its conjugate?
4. If the denominator of a fraction is a complex number in real and imaginary part, one should multiply its conjugate to both the numerator and denominator. The denominator will become a real number as it is multiplied by its conjugate. This would allow you to further simplify the division.