Mechanical Vibrations 4600: 431 **Homework 06**
Due: August 14, 2007

Name: _____

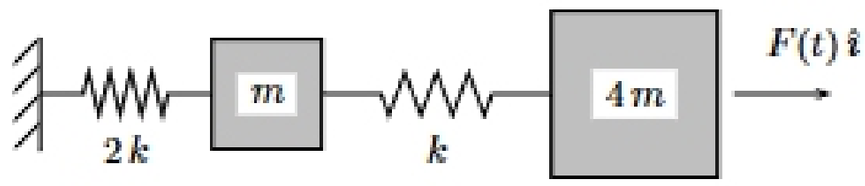
Please denote your answers clearly, i.e., box in, star, etc., and *write neatly*. **There are no points for small, messy, unreadable work**... please use lots of paper.

Problem 1:
For the system shown to the right

a) use Lagrange's equations to determine the equations of motion;

b) find the mode shapes and natural frequencies of the system;

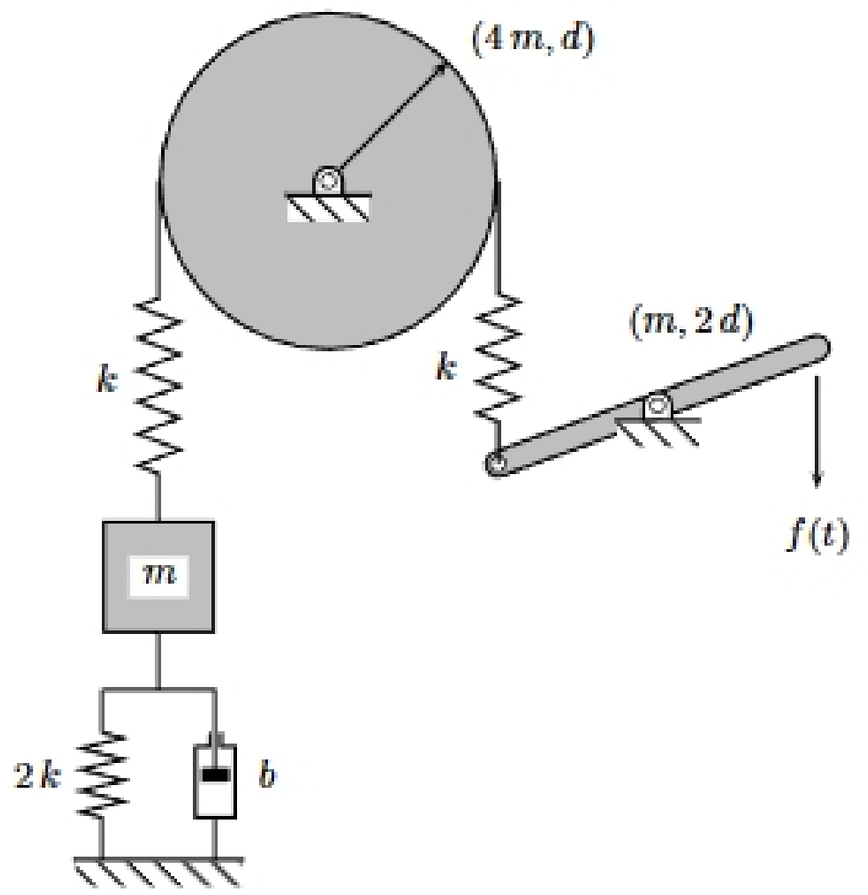
c) determine the modal equations for this system.



Problem 2:
For the system shown to the right

a) find the equations of motion;

b) are your equations statically coupled, dynamically coupled, both or neither?

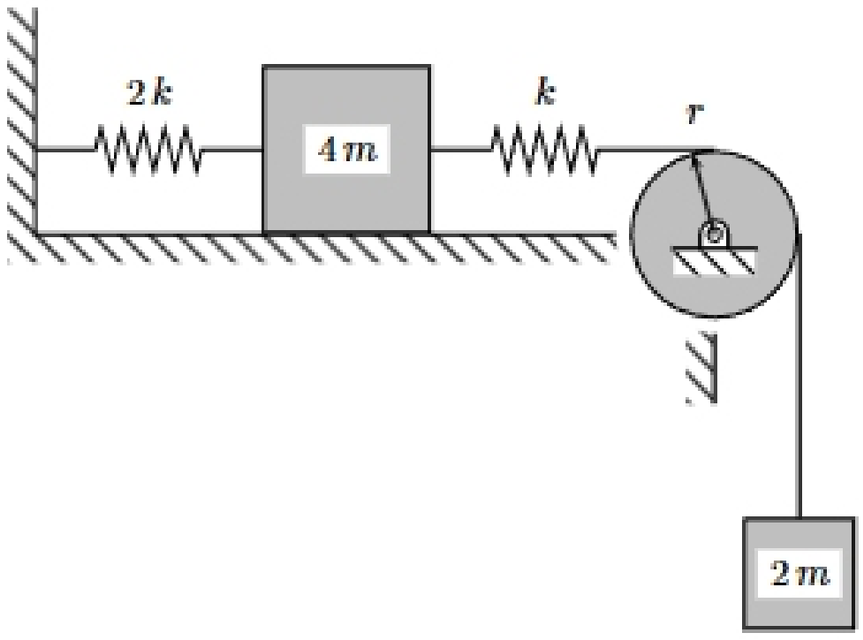


Problem 3:
In the multi-degree-of-freedom system shown in the figure, the block with mass $4m$ slides on a smooth, frictionless surface. If the pulley is massless:

a) using Lagrange's equations, determine the differential equations governing the motion, as measured from static equilibrium;

b) with $m = 1$ kg and $k = 16$ N/m, find the natural frequencies and mode shapes for the free vibration of this system. Normalize the mode shapes so that with respect to the mass matrix the amplitude of each mode is one;

c) find the general solution to these equations for the above values of m and k.

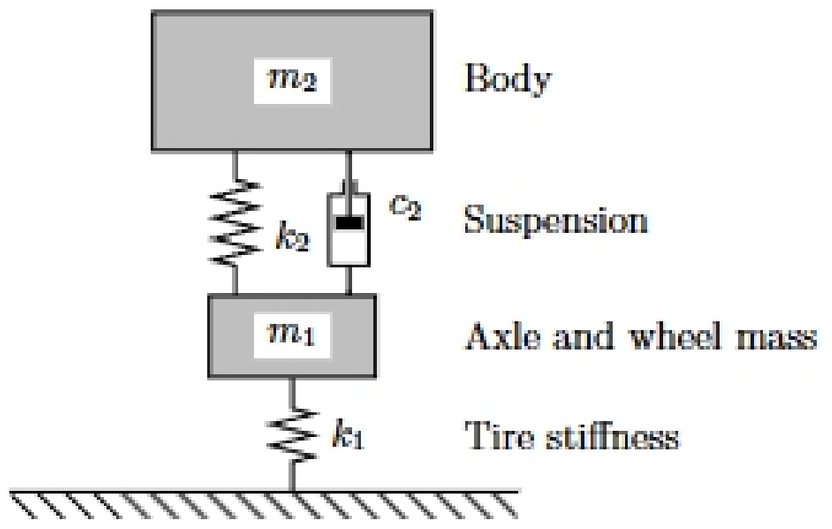


Problem 4:
To describe the vertical motions of an automobile, the two degree-of-freedom system shown in the figure, known as a *quarter-car model*, is used.

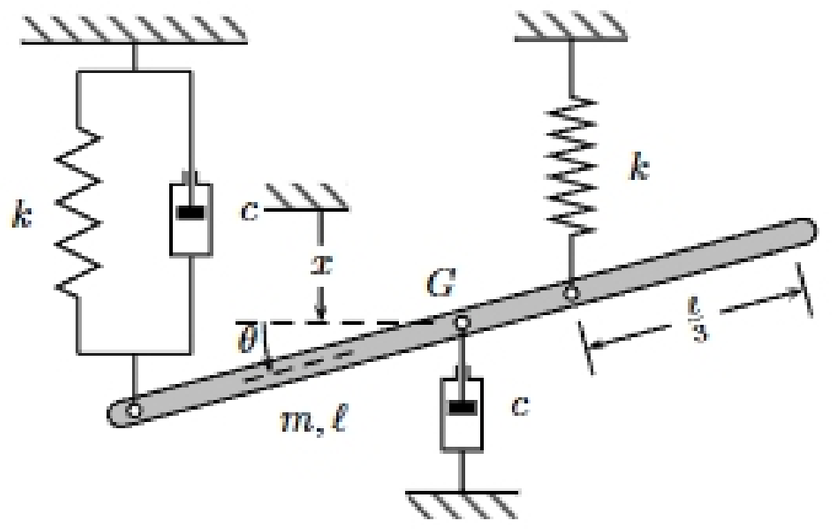
a) Find the equations of motion for this system.

b) In the absence of damping, with $c_2 = 0$, find the natural frequencies and mode shapes of the response with the parameters

$$m_1 = 80 \text{ kg}, \qquad m_2 = 1100 \text{ kg},$$
$$k_1 = 300 \text{ kN/m}, \qquad k_2 = 30 \text{ kN/m}.$$



Problem 5:
For the mechanical system shown to the right, the uniform rigid bar is supported by identical springs and dampers. For this system find the equations of motion in terms of x and θ.



Problem 6:
For the mechanical system shown to the right, the uniform rigid bar with mass m and length ℓ is supported by identical springs while a block of mass $2m$ is suspended from the bar. For this system

a) find the equations of motion in terms of x, z, and θ.

b) if $k = 6400$ N/m, $m = 8$ kg, and $\ell = 0.50$ m, and the natural frequencies of the system are

$$\omega_1 = 15.65 \text{ rad/s}, \quad \omega_2 = 48.48 \text{ rad/s},$$
$$\omega_3 = 77.50 \text{ rad/s},$$

find the corresponding mode shapes for this system;