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AAE 340 Homework Solutions #6

(See Lecture 18 notes for more detail on problem setup)

Problem #1: Galileo orbiter and probe arrival @ Jupiter

Given:

$r_0 = 300 R_J$ (Initial radius of probe & orbiter)

$V_\infty = 5.455$ km/s (orbiter & probe)

$\mu = 1.267 \times 10^8$ km^3/s^2 (gravitational parameter of Jupiter)

$R_J = 71398$ km (radius of Jupiter)

$P = 198$ days (for orbiter's elliptic orbit, following Jupiter insertion maneuver)

$r_p = 4 R_J$ (for orbiter)

$r_p = 1 R_J$ (for probe)

Find:

1) EOM in state variable form; outline of MATLAB

The EOMs are:

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2}$$

$$\ddot{\theta} + \frac{2\dot{r}\dot{\theta}}{r} = 0$$

Define state variables:

$$x_1 = r; \quad x_2 = \dot{r}; \quad x_3 = \theta; \quad x_4 = \dot{\theta}$$

State space representation of the EOMs:

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 x_4^2 - \frac{\mu}{x_1^2} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -\frac{2 x_2 x_4}{x_1} \end{aligned}}$$

The required MATLAB program will:

1) Contain the state eqns. in a function file to be called by the ode 45 numerical integrator.

2) perform 3 different numerical integrations – one each for:

- hyperbolic orbiter trajectory
- hyperbolic probe trajectory
- elliptic orbiter trajectory

3) Use the appropriate set of ICs $(r_0, \dot{r}_0, \theta_0, \dot{\theta}_0)$ for the three numerical integrations

2 Find the Initial Conditions

a) Hyperbolic Orbiter Trajectory

$$\boxed{r(0) = r_0 = 300\, R_J = 2.1419 \times 10^7 \text{ km}}$$

Now, using Conservation of Specific Energy:

$$\mathcal{E} = \frac{1}{2} v_0^2 - \frac{\mu}{r_0} = \frac{1}{2} V_\infty^2 \quad \text{for hyperbolic orbit}$$

$$\Rightarrow \quad V_0 = \sqrt{V_\infty^2 + 2\frac{\mu}{r_0}}$$

$$V_0 = 6.4488 \text{ km/s}$$

(Note: if r_0 was sufficiently large, then $V_0 \approx V_\infty$. For $r_0 = 300\,R_J$, V_0 + V_∞ differ by $\sim 1 \frac{km}{s}$)

Note (Important): Find your final answer good to 4 sig. figures because $V_\infty = 5.455$ km/s limits the precision. But carry 5 figures to maintain accuracy in subsequent calculations (i.e. to avoid round-off errors). Note the difference between the 1) sig. figs. (how precisely do we know the value) and 2) precision of calculations (to avoid loss of sig. figs. in the final answer).

Now, using conservation of specific angular momentum:

$$h = r^2 \dot{\theta} = r v \cos \gamma \qquad \gamma: \text{flight path angle}$$

@ perijove (Jupiter periapsis) $\gamma = 0 \Rightarrow h = r_p v_p$

3/18

We know that r_p for the orbiter is $4R_J$, using $\mathcal{E}$ to find v_p :

$$v_p = \sqrt{V_\infty^2 + 2\frac{\mu}{r_p}}$$

$$v_p = 30.2826 \text{ km/s}$$

Going back to $h = r_p v_p$:

$$h = (4.71398 \text{ km})(30.2826 \text{ km/s})$$
$$h = 8.64847 \times 10^6 \text{ km}^2/\text{s}$$

Use $h = r_p v_p = r_0 v_0 \cos\gamma_0$ to find initial flight path angle (γ_0)

$$\gamma_0 = \cos^{-1}\left(\frac{r_p v_p}{r_0 v_0}\right) = \cos^{-1}\left(\frac{8.64847 \times 10^6 \text{ km}^2/\text{s}}{(2.1419 \times 10^7 \text{ km})(6.4488 \text{ km/s})}\right)$$

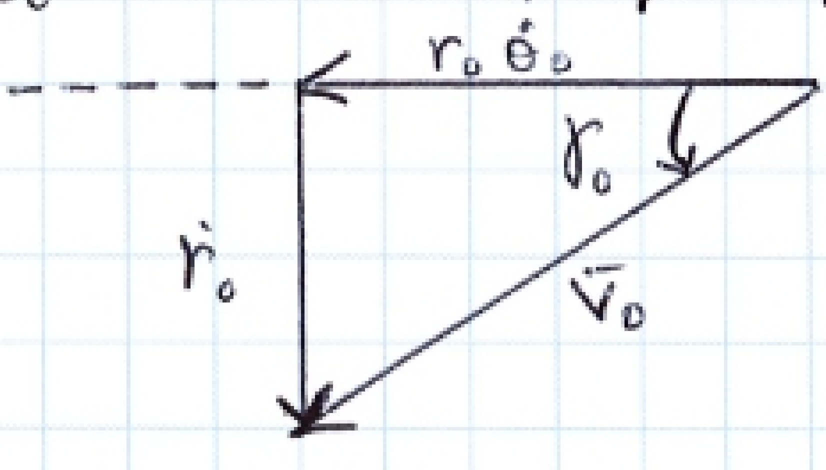
$$\gamma_0 = \pm 86.4103°$$

Choose <u>negative</u> value since the orbiter is <u>approaching</u> Jupiter

$$\Rightarrow \gamma_0 = -86.4103° \quad (-1.5081 \text{ rad})$$

Now, resolve the velocity vector of the orbiter @ $t=0$ into its components to find $\dot{r}(0)$ and $\dot{\theta}(0)$.

Recall : ${}^I\bar{v}^{OP} = \dot{r}\,\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta$



$$\sin\gamma_0 = \frac{\dot{r}_0}{v_0}$$

$$\cos\gamma_0 = \frac{r_0\dot{\theta}_0}{v_0}$$

Alternate Method:
Get $\dot{\theta}_0$ from $h = r_0^2\dot{\theta}_0$
Then, get $\dot{r}_0$ from
$v_0^2 = \dot{r}_0^2 + r_0^2\dot{\theta}_0^2$
$\Rightarrow \dot{r}_0 = \overset{+}{\ominus}\sqrt{v_0^2 - r_0^2\dot{\theta}_0^2}$

$$\Rightarrow \dot{r}_0 = v_0 \sin\gamma$$

solution is <u>negative</u> since we are approaching Jupiter, and r must be <u>decreasing</u>.

$$\boxed{\dot{r}_0 = -6.436 \text{ km/s}}$$

$$\Rightarrow \dot{\theta}_0 = \frac{v_0}{r_0}\cos\gamma_0$$

$$\boxed{\dot{\theta}_0 = 1.885 \times 10^{-8} \text{ rad/s}}$$

4 sig. figs.
(we did not loose or gain accuracy)